EPFL

Aperture Antennas

Antennas

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Antennas

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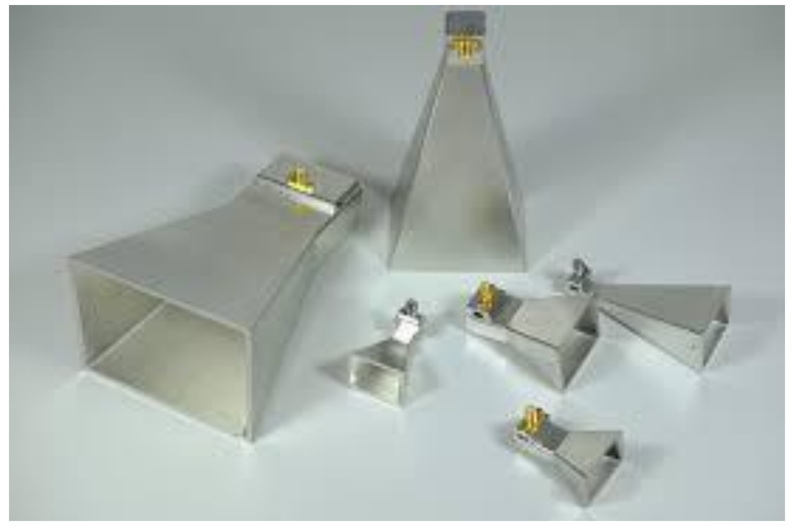
Lecture Outline

- What is an aperture?
- Recall: field equivalence principle
- Rectangular aperture
- Uniform Distribution on an Infinite Ground Plane
- Distribution Comparison on an Infinite Ground Plane
- Horn Antenna
  - Pyramidal Horn
  - Conical Horn
  - Corrugated Horn
  - Ridged Horn

Aperture Antenna

An ***aperture antenna*** usually refers to a (metallic) sheet with a hole (or an aperture) of some shape through which radiation comes out. These antennas are characterized by using an ***opening*** or ***closed surface*** as the radiating element, allowing for the transmission or reception of electromagnetic waves. They are commonly used in applications such as radar, satellite communication, and aircraft systems due to their ability to be mounted on surfaces.



Open Waveguide

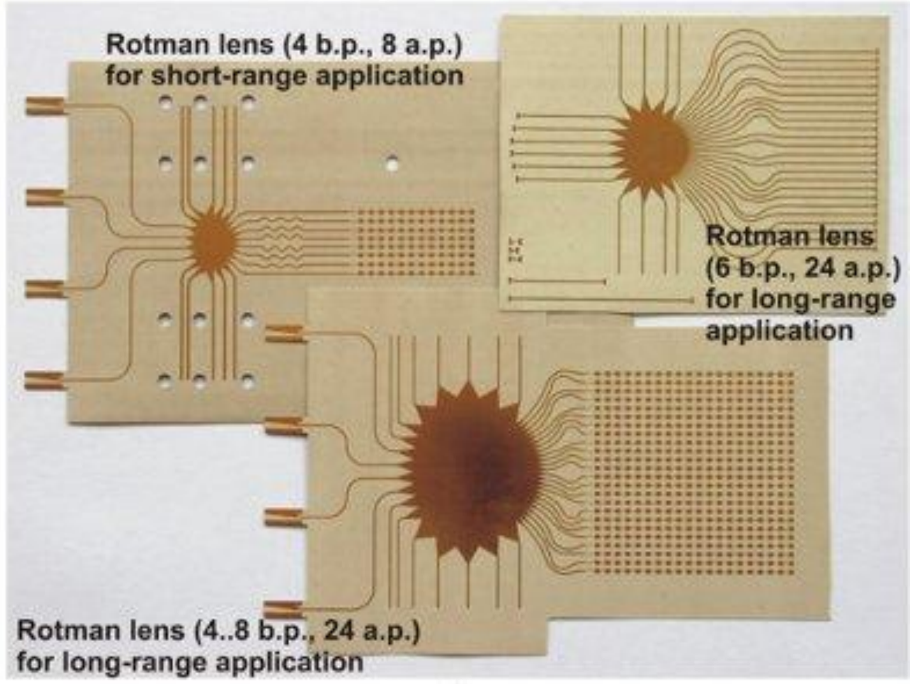
Horn Antennas

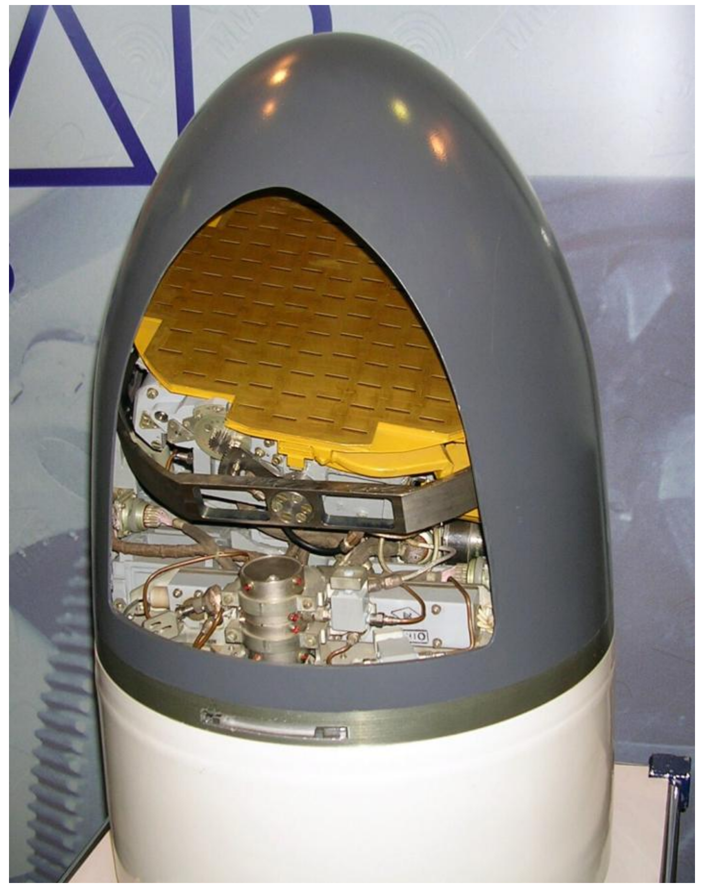
Slot Antennas

Reflector Antennas

Microstrip Antennas

Lens Antennas





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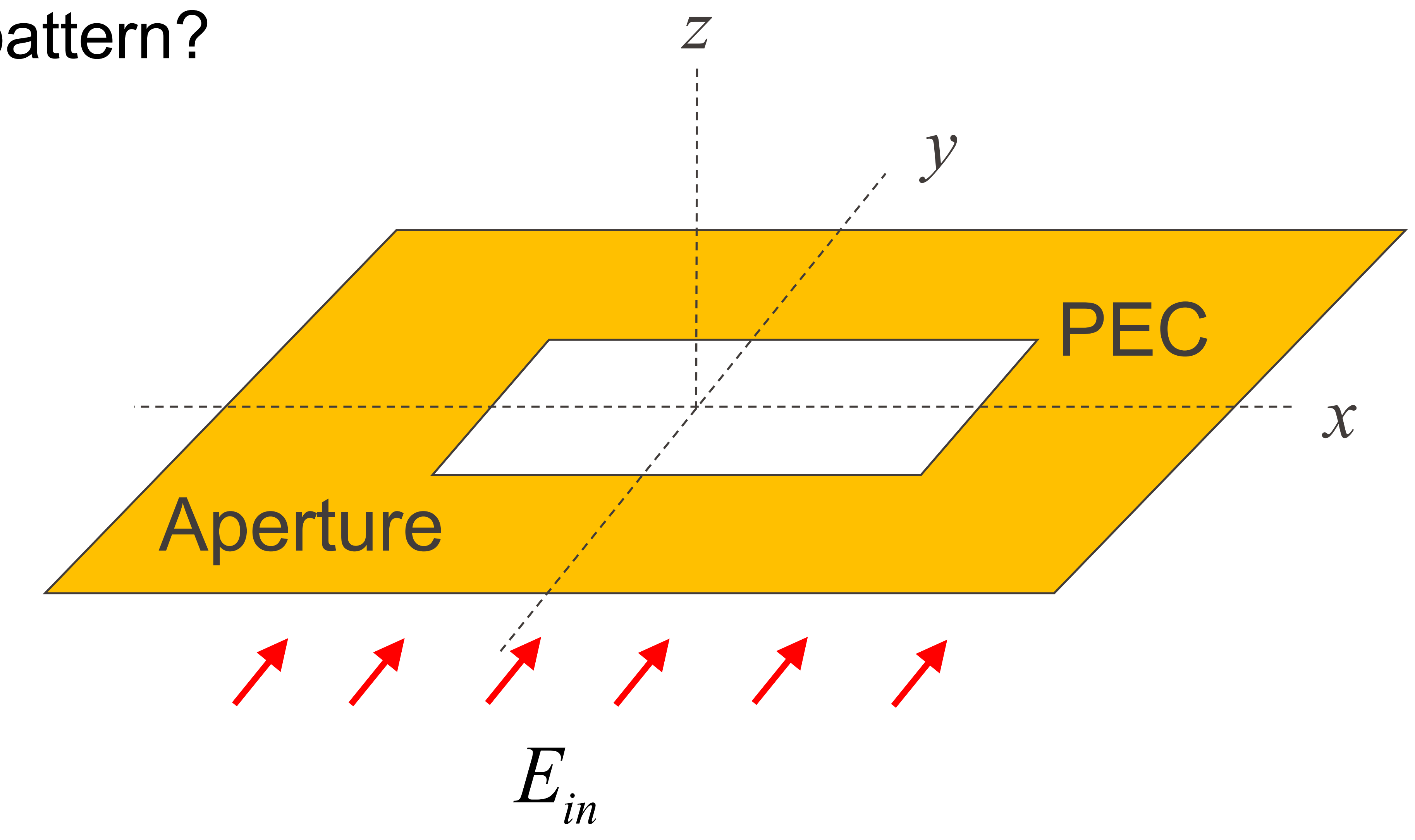
Aperture Antenna

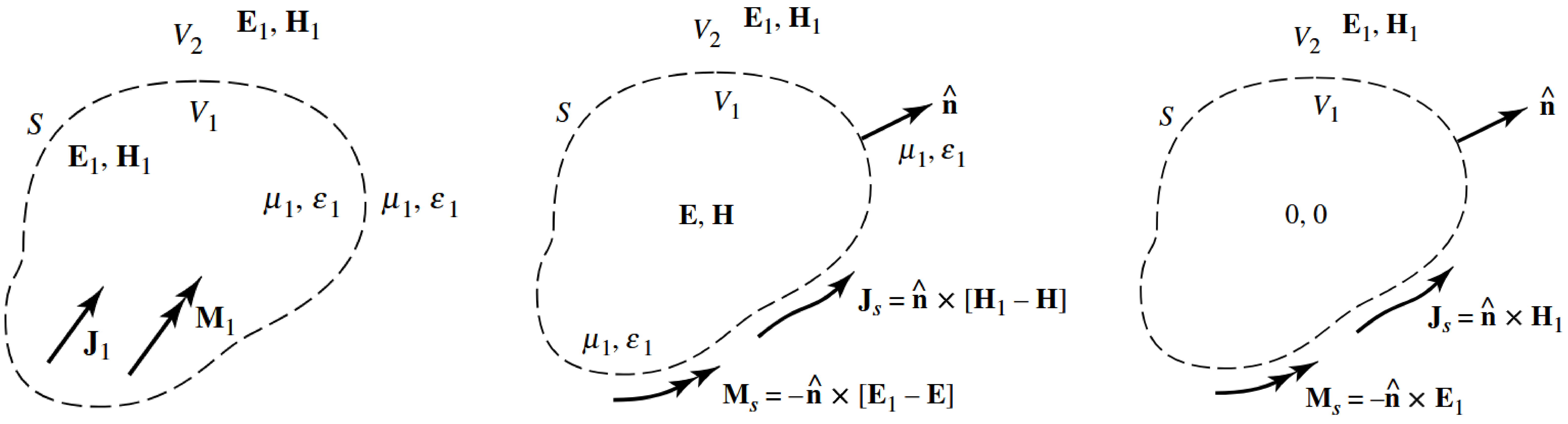
Questions: Considering an impinging wave at the aperture, E_{in}

- What happens on the other side?
- How does the radiation come out of the aperture looks like when the dimensions of the hole are of the order of the wavelength?.
- What is the radiation pattern?



Aperture Antenna

Recall: Field Equivalence Principle (Huygens' Principle)

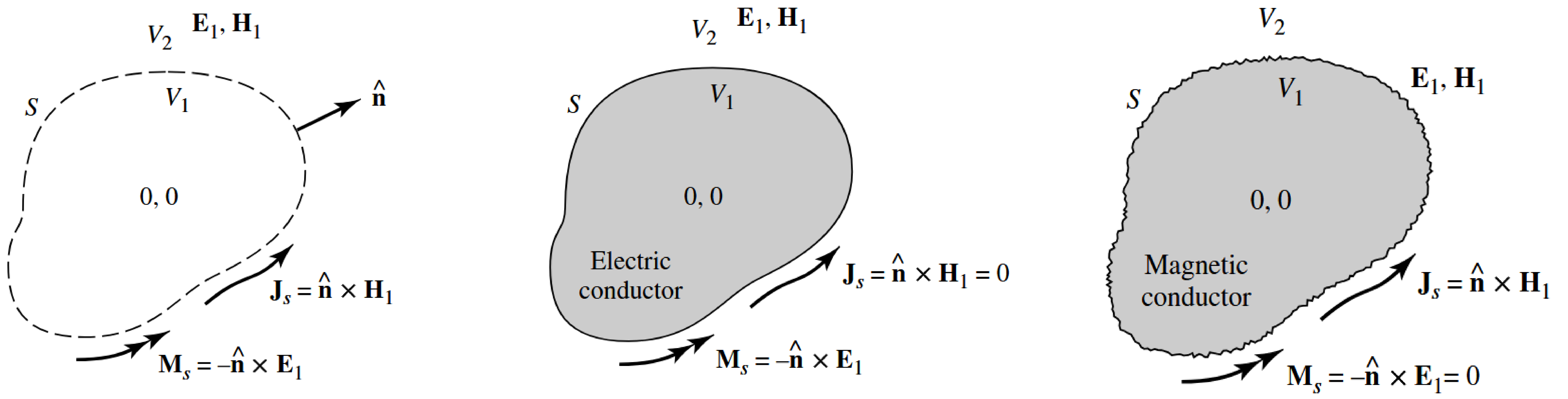


Actual problem Equivalent problem (Huygens) Love's equivalent

Since the fields ***E***, ***H*** within *S* can be anything (this is not the region of interest), it can be assumed that they are zero.

Aperture Antenna

Recall: Field Equivalence Principle (Huygens' Principle)

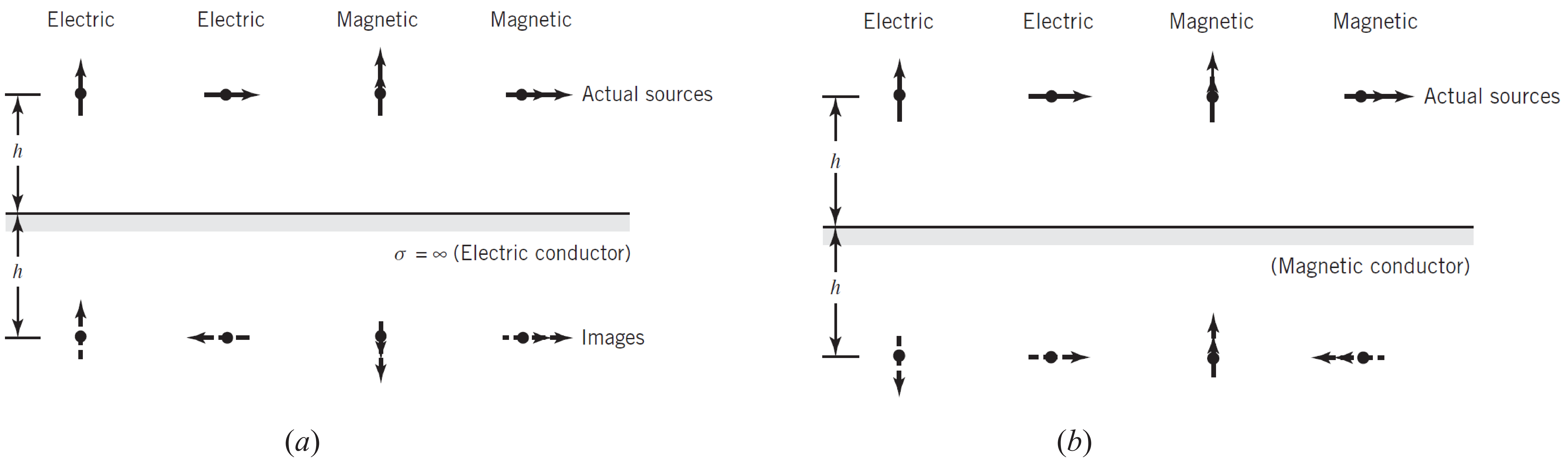


Love's equivalent   Electric conductor equivalent   Magnetic conductor equivalent

The equivalence principle states that the surface *S* can be replaced either by a perfect electric conductor (PEC) or a perfect magnetic conductor (PMC). By enforcing the corresponding boundary condition, one of the equivalent currents vanishes, and the problem can be solved using only the remaining surface current.

Aperture Antenna

Recall: The method of images

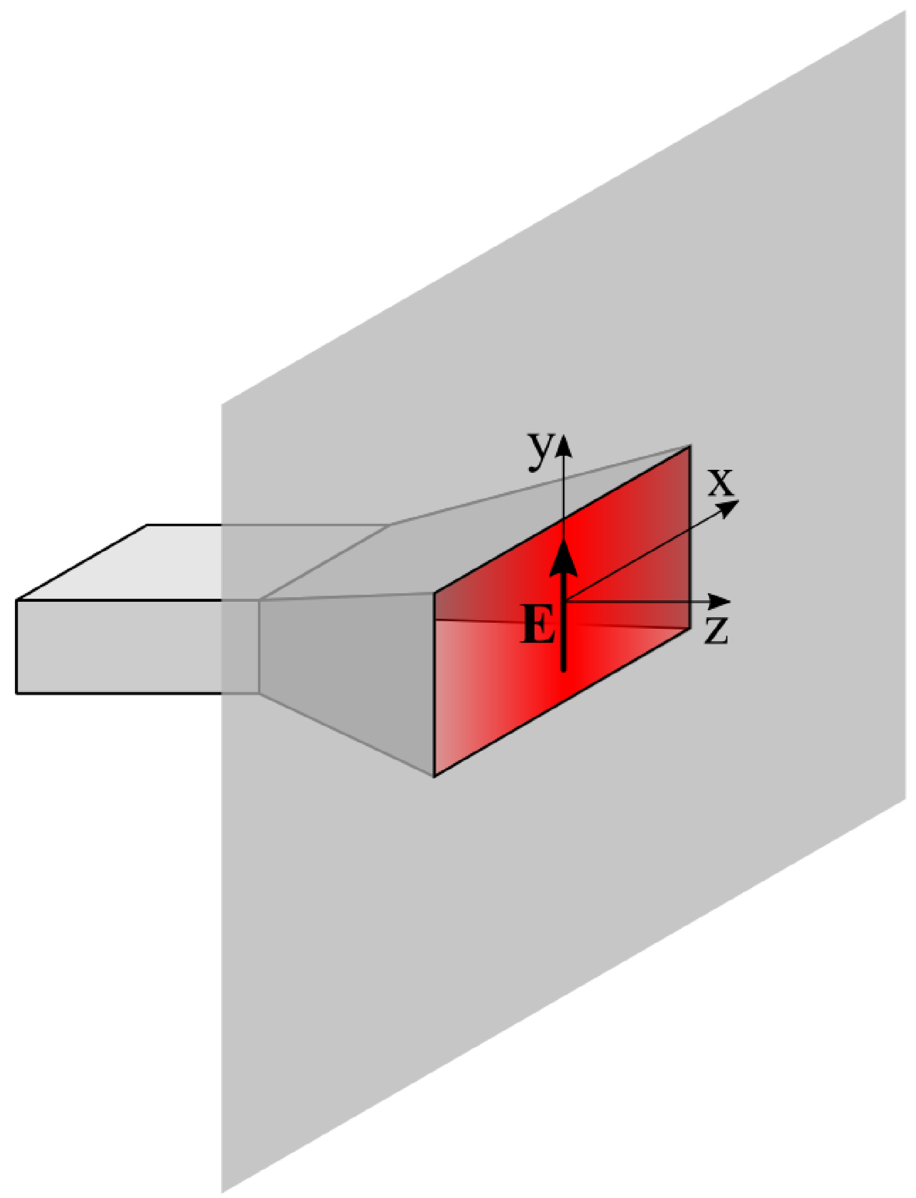


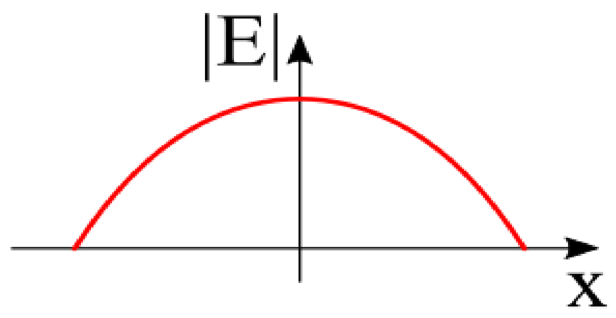
Electric and magnetic sources and their images near (a) electric and (b) magnetic conductors

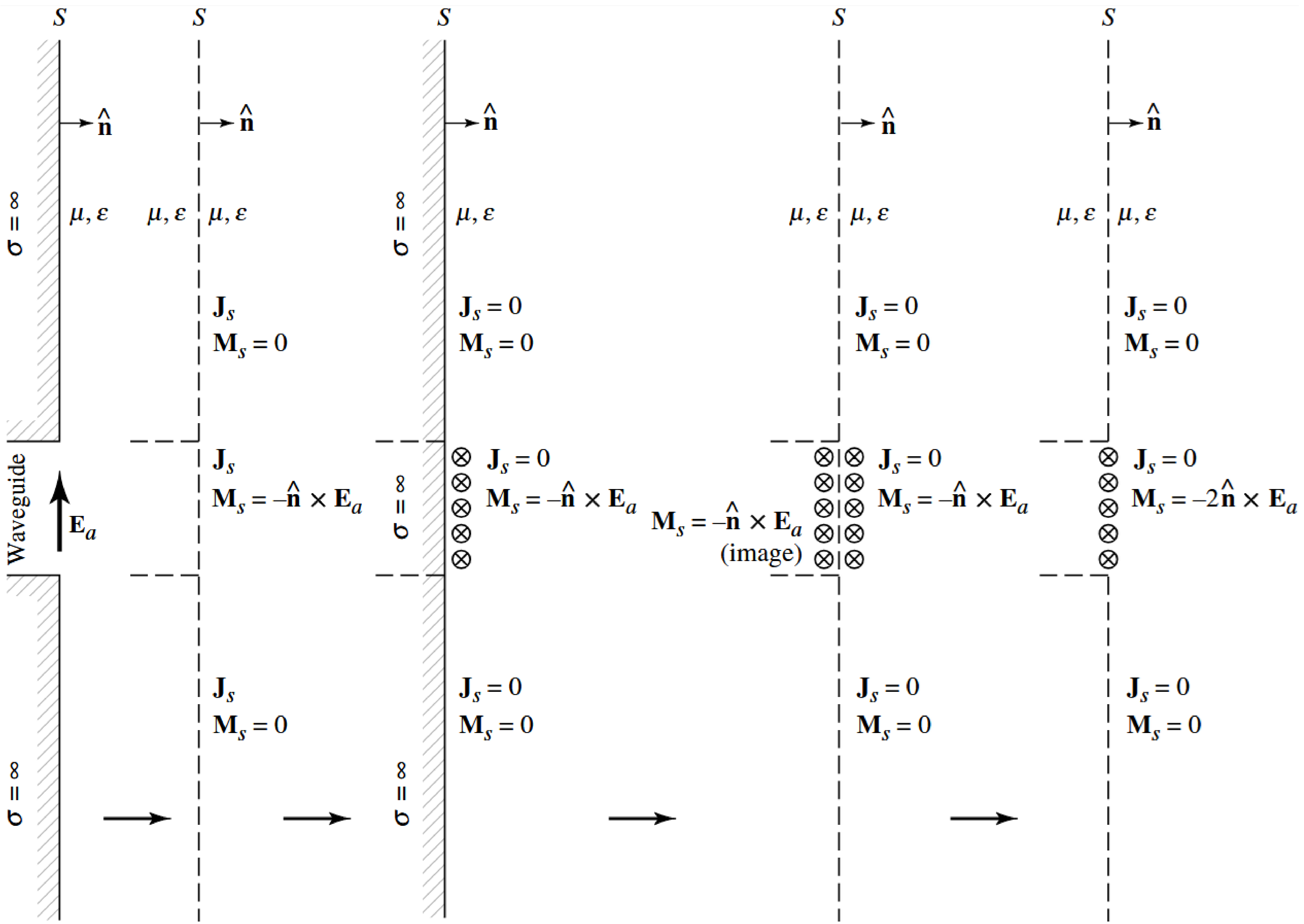
Rectangular Aperture

Field Equivalence Principle (Huygens' Principle)

Assume a waveguide connected to an infinite PEC plan with aperture







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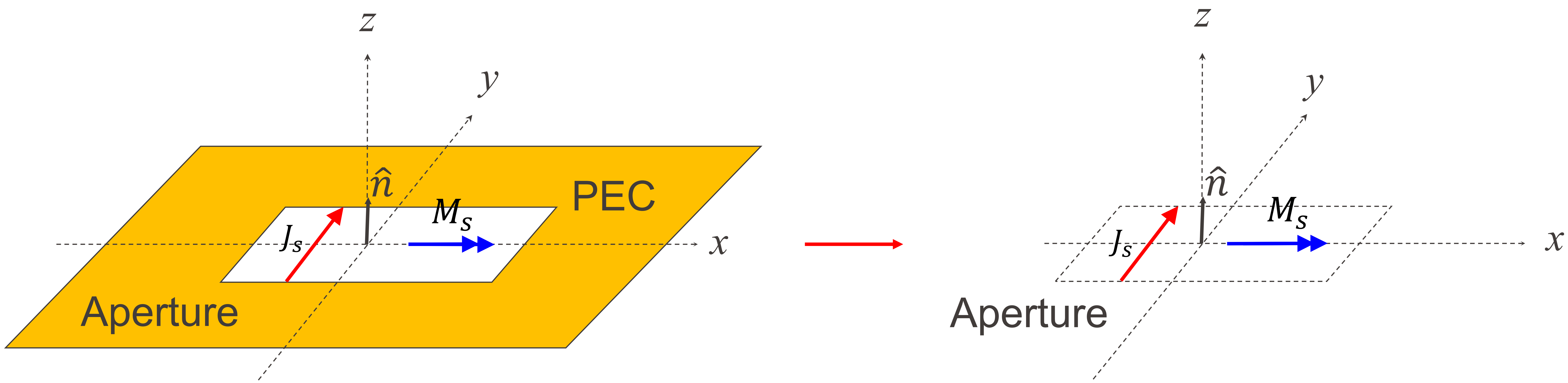
Rectangular Aperture

Electric & Magnetic Current

Fields radiated by sources $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ in an unbounded medium can be computed by using ***A*** and ***F*** vector potentials, where the integration must be performed over the entire surface occupied by $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$. These equations yield valid solutions for all observation points.

$$\boldsymbol{J_s} = \hat{n} \times H_a \qquad \boldsymbol{M_s} = -\hat{n} \times E_a$$



$$\boldsymbol{J_s} = 0 \qquad \boldsymbol{M_s} = -\hat{n} \times E_a$$

So, let's find the far-field...

Rectangular Aperture

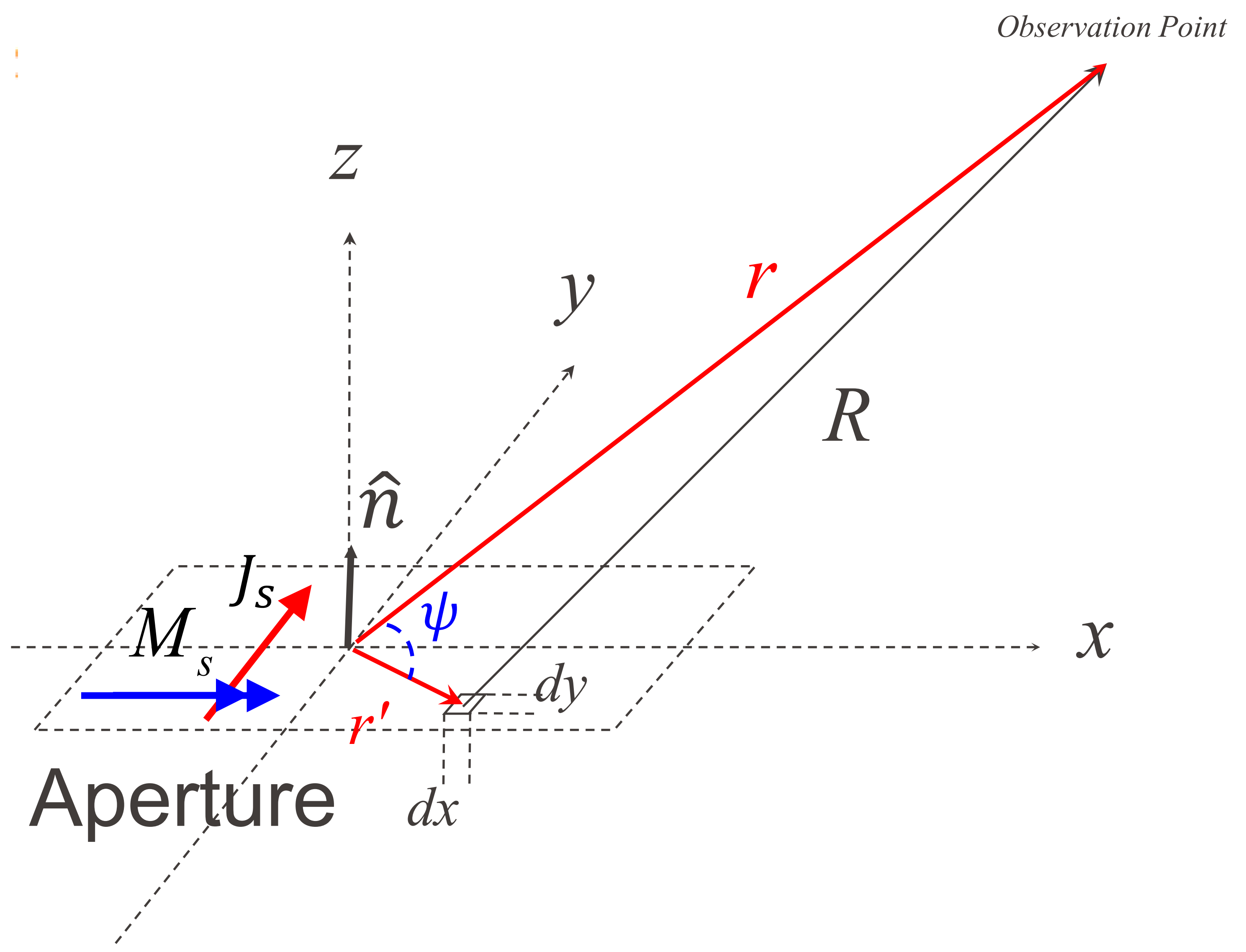
Electric & Magnetic Vector Potential

In general form, the fields radiated by sources $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$ in an unbounded medium.

$$\mathbf{A} = \frac{\mu}{4\pi}\iint_S \mathbf{J}_s \frac{e^{-jkR}}{R}\, ds' \qquad \mathbf{F} = \frac{\epsilon}{4\pi}\iint_S \mathbf{M}_s \frac{e^{-jkR}}{R}\, ds'$$

Since

$$\vec{r} = \overrightarrow{r'} + \vec{R} \longrightarrow |\vec{r}| \approx |\overrightarrow{r'}| cos\psi + |\vec{R}|$$



For ***far-field*** observations *R* can most commonly be approximated by

$$R \simeq r - r' \cos\psi \quad \text{for phase variations}$$
$$R \simeq r \quad \text{for amplitude variations}$$

where ψ is the angle between the vectors *r* and *r*′. The primed coordinates (x', y', z', or r', θ', φ') indicate the space occupied by the sources $\boldsymbol{J_s}$ and $\boldsymbol{M_s}$, over which integration must be performed.

N- and L-functions

Recall: N- and L-functions

Using the rectangular-to-spherical component transformation, obtained by taking the inverse, they reduce for the θ and φ components to

$$\mathbf{A} = \frac{\mu}{4\pi} \iint_S \mathbf{J}_s \frac{e^{-jkR}}{R}\, ds' \simeq \frac{\mu e^{-jkr}}{4\pi r}\mathbf{N}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr'\cos\psi}\, ds'$$

$$\mathbf{F} = \frac{\epsilon}{4\pi} \iint_S \mathbf{M}_s \frac{e^{-jkR}}{R}\, ds' \simeq \frac{\epsilon e^{-jkr}}{4\pi r}\mathbf{L}$$

$$\mathbf{L} = \iint_S \mathbf{M}_s e^{jkr'\cos\psi}\, ds'$$

where

$$N_\theta = \iint_S [J_x \cos\theta\cos\phi + J_y \cos\theta\sin\phi - J_z \sin\theta] e^{+jkr'\cos\psi}\, ds'$$

$$N_\phi = \iint_S [-J_x \sin\phi + J_y \cos\phi] e^{+jkr'\cos\psi}\, ds'$$

$$L_\theta = \iint_S [M_x \cos\theta\cos\phi + M_y \cos\theta\sin\phi - M_z \sin\theta] e^{+jkr'\cos\psi}\, ds'$$

$$L_\phi = \iint_S [-M_x \sin\phi + M_y \cos\phi] e^{+jkr'\cos\psi}\, ds'$$

E- and H- Far-fields

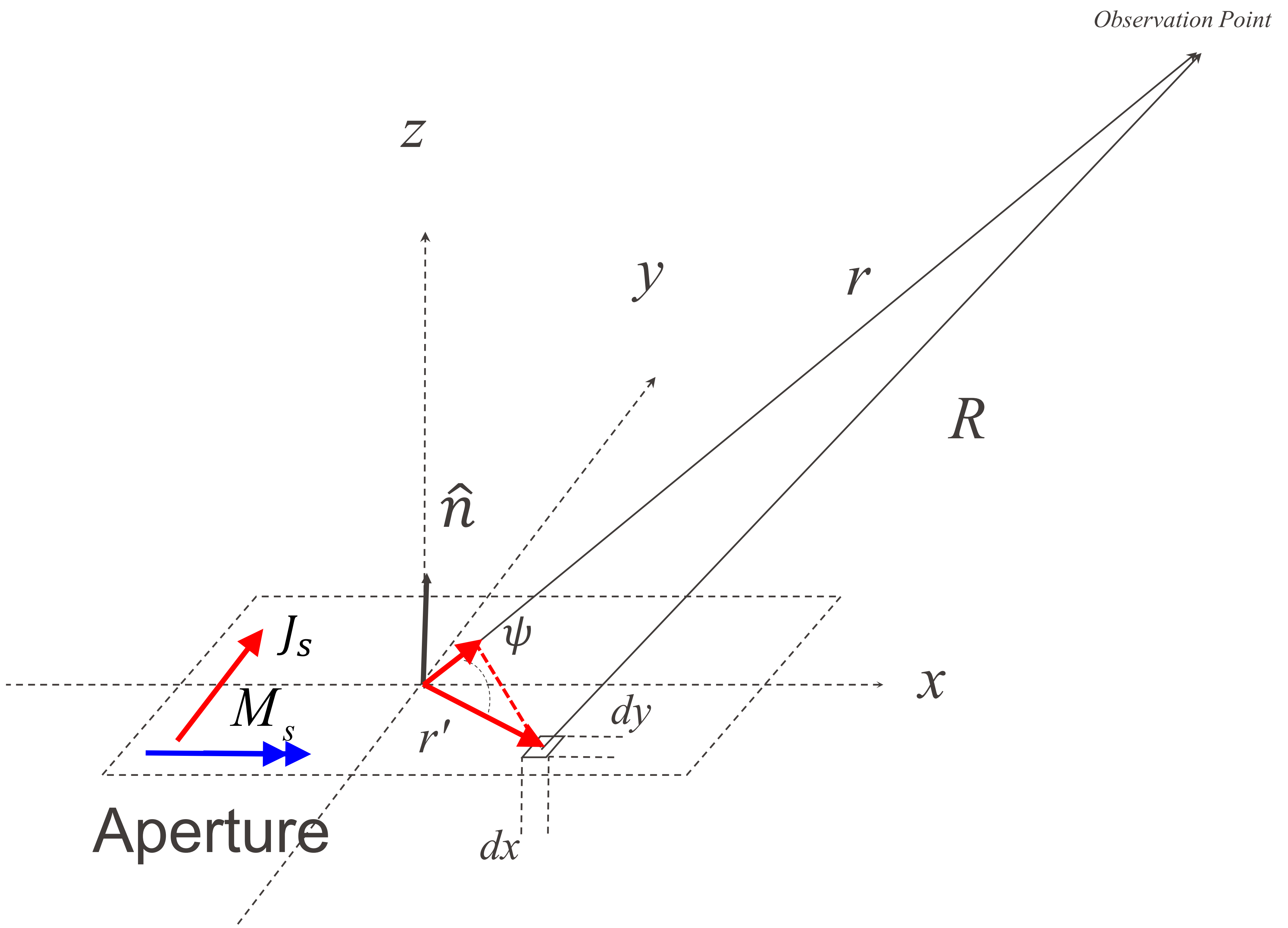
The differential paths take the form of

$$\mathbf{A} = \frac{\mu}{4\pi} \iint_S \mathbf{J}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\mu e^{-jkr}}{4\pi r} \mathbf{N}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr' \cos\psi} ds'$$

$$\mathbf{F} = \frac{\epsilon}{4\pi} \iint_S \mathbf{M}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\epsilon e^{-jkr}}{4\pi r} \mathbf{L}$$

$$\mathbf{L} = \iint_S \mathbf{M}_s e^{jkr' \cos\psi} ds'$$



$$r' cos\psi \approx \overrightarrow{r'} \cdot \hat{r} = (\hat{x}x' + \hat{y}y') \cdot (\hat{x}sin\theta cos\phi + \hat{y}sin\theta sin\phi + \hat{z}cos\theta)$$
$$= x' sin\theta cos\phi + y' sin\theta sin\phi$$

E- and H- Far-fields

Combining these equations the total ***E***- and ***H***-fields can be written as

$$E_r \simeq 0$$

$$E_\theta \simeq -\frac{jke^{-jkr}}{4\pi r}(L_\phi + \eta N_\theta)$$

$$E_\phi \simeq +\frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\phi)$$

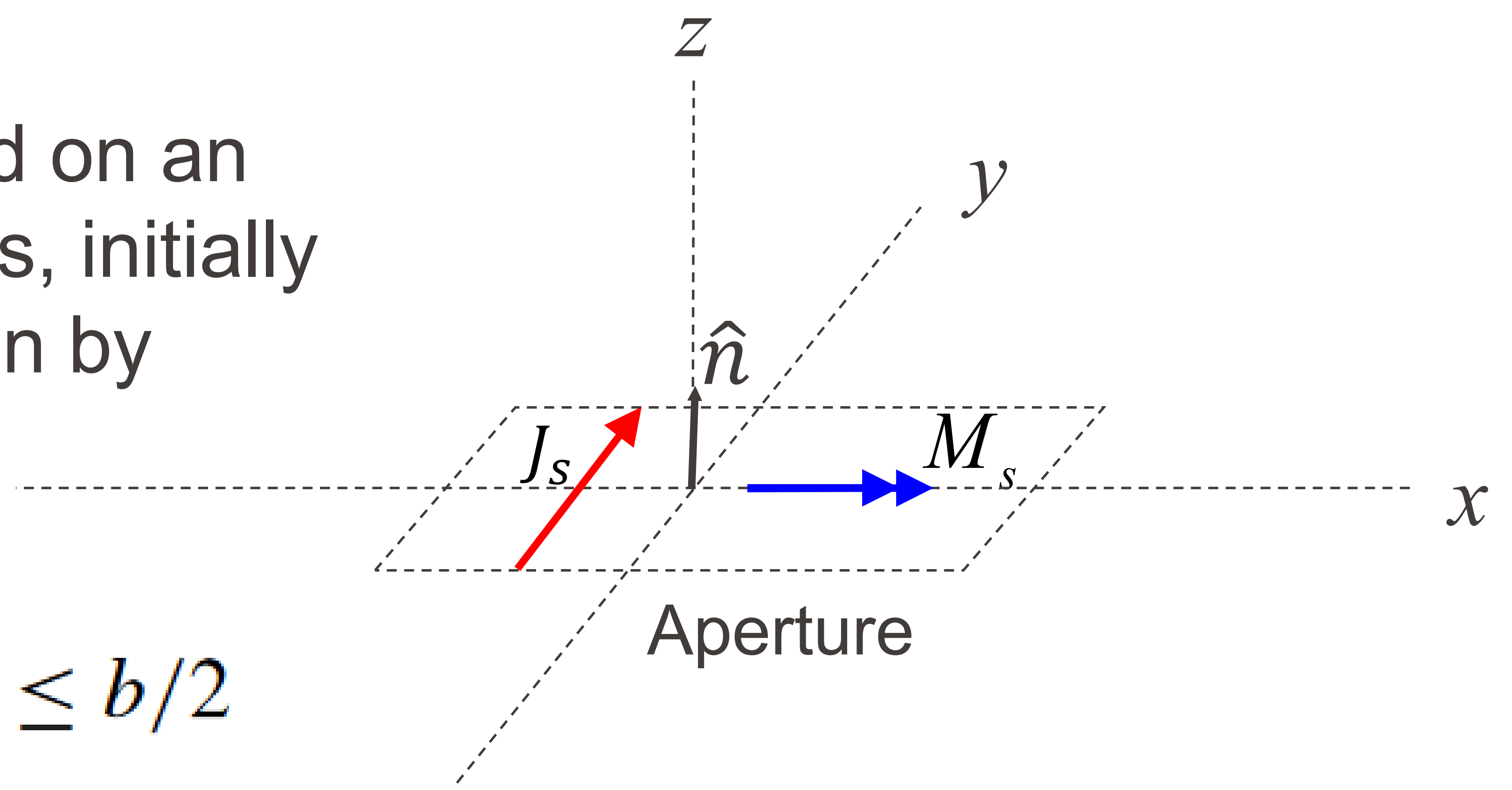
$$H_r \simeq 0$$

$$H_\theta \simeq \frac{jke^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)$$

$$H_\phi \simeq -\frac{jke^{-jkr}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right)$$

Uniform Distribution on an Infinite Ground Plane

The first aperture examined is a *rectangular aperture* mounted on an *infinite ground plane*. To reduce the mathematical complexities, initially the field over the opening is assumed to be constant and given by



$$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \quad -a/2 \le x' \le a/2, \quad -b/2 \le y' \le b/2$$

and E_0 is a constant. The task is to find the fields radiated by it, the pattern beamwidths, the side lobe levels of the pattern, and the directivity. To accomplish these, the equivalent will be formed first.

$$\overrightarrow{M_s^e} = -2\hat{n} \times \overrightarrow{E_a} = -2\hat{z} \times \overrightarrow{E_y} = 2\hat{x} \times E_0 \longrightarrow \overrightarrow{M_x^e} \qquad \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix}$$

$$\overrightarrow{J_s^e} = \hat{n} \times \vec{H} \approx 0$$

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Uniform Distribution on an Infinite Ground Plane

Recall: Vector Potential & Far-field Radiation

$$\mathbf{F} = \frac{\epsilon}{4\pi} \iint_S \mathbf{M}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\epsilon e^{-jkr}}{4\pi r} \mathbf{L}$$

$$\mathbf{L} = \iint_S \mathbf{M}_s e^{jkr' \cos\psi} ds'$$

$$\mathbf{A} = \frac{\mu}{4\pi} \iint_S \mathbf{J}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\mu e^{-jkr}}{4\pi r} \mathbf{N}$$

$$\mathbf{N} = \iint_S \mathbf{J}_s e^{jkr' \cos\psi} ds'$$

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Uniform Distribution on an Infinite Ground Plane

Recall: Vector Potential & Far-field Radiation

Since

$$\overrightarrow{M_s^e} = -2\hat{n} \times \overrightarrow{E_a} = -2\hat{z} \times \overrightarrow{E_y} = 2\hat{x} \times E_0 \longrightarrow \overrightarrow{M_x^e}$$

$$\overrightarrow{J_s^e} = \hat{n} \times \vec{H} \approx 0 \longrightarrow \mathbf{N} = 0$$

Thus

$$L_\theta = \iint_S [M_x \cos\theta \cos\phi + \cancel{M_y} \cos\theta \sin\phi - \cancel{M_z} \sin\theta] e^{+jkr' \cos\psi}\, ds'$$

$$L_\phi = \iint_S [-M_x \sin\phi + \cancel{M_y} \cos\phi] e^{+jkr' \cos\psi}\, ds'$$

Uniform Distribution on an Infinite Ground Plane

N- and L-functions

The far-zone fields radiated by the aperture can be found by

$$L_\theta = \int_{-b/2}^{+b/2} \int_{-a/2}^{+a/2} [M_x \cos\theta \cos\phi] e^{jk(x' \sin\theta \cos\phi + y' \sin\theta \sin\phi)} \, dx' \, dy'$$

$$\longrightarrow \quad L_\theta = \cos\theta \, \cos\phi \left[\int_{-b/2}^{+b/2} \int_{-a/2}^{+a/2} M_x e^{jk(x' \sin\theta \cos\phi + y' \sin\theta \sin\phi)} \, dx' \, dy' \right]$$

Using the integral

$$\int_{-c/2}^{+c/2} e^{j\alpha z} \, dz = c \left[\frac{\sin\left(\frac{\alpha}{2} c\right)}{\frac{\alpha}{2} c} \right]$$

It reduces to

$$L_\theta = 2abE_0 \left[\cos\theta \cos\phi \left(\frac{\sin X}{X} \right) \left(\frac{\sin Y}{Y} \right) \right]$$

where

$$X = \frac{ka}{2} \sin\theta \cos\phi$$

$$Y = \frac{kb}{2} \sin\theta \sin\phi$$

Similarly it can be shown that

$$L_\phi = -2abE_0 \left[\sin\phi \left(\frac{\sin X}{X} \right) \left(\frac{\sin Y}{Y} \right) \right]$$

Uniform Distribution on an Infinite Ground Plane

Substituting L_θ and L_φ, the fields radiated by the aperture can be written as

$$E_r = 0$$

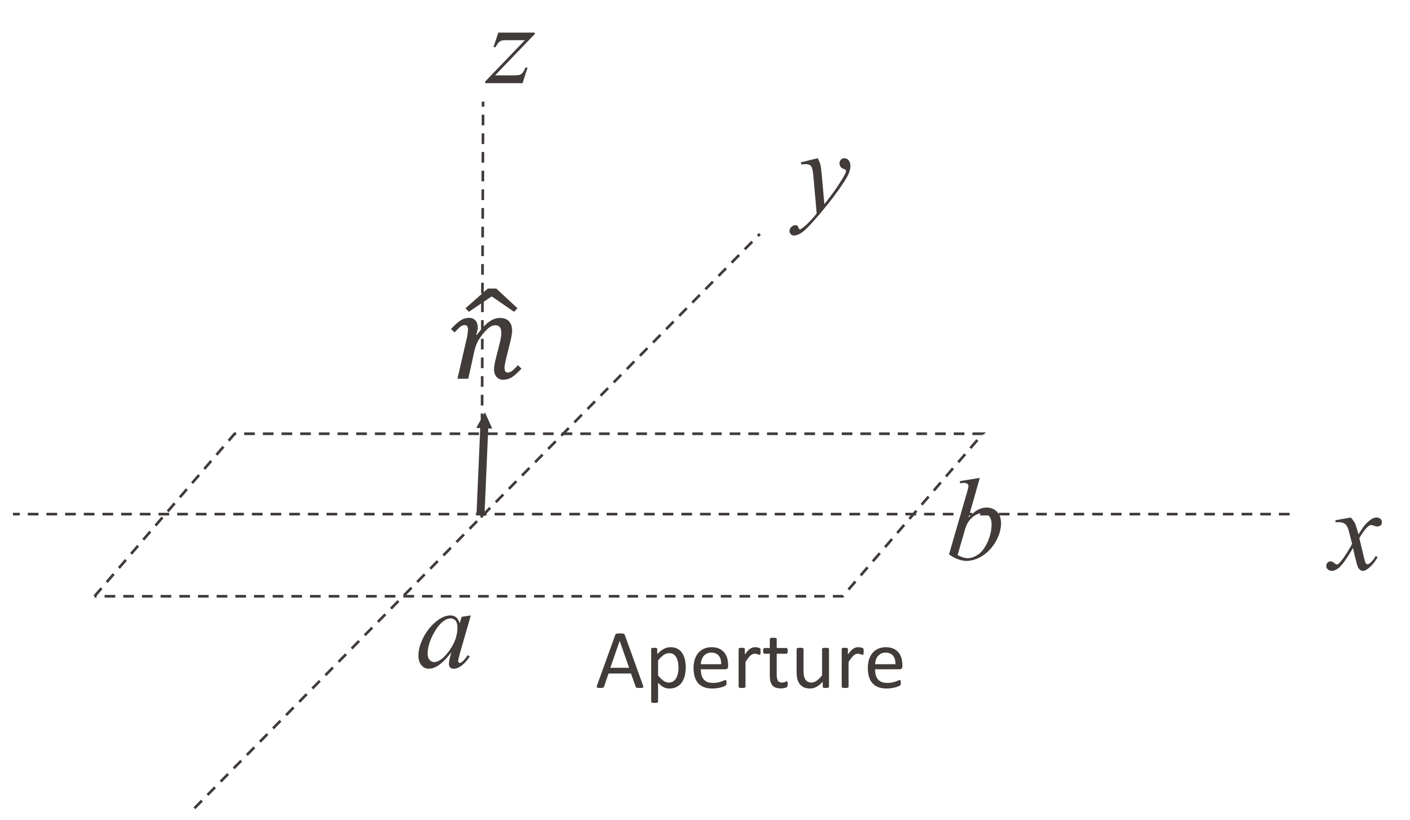
$$E_\theta = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left[\sin\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

$$E_\phi = j\frac{abkE_0 e^{-jkr}}{2\pi r}\left[\cos\theta\cos\phi\left(\frac{\sin X}{X}\right)\left(\frac{\sin Y}{Y}\right)\right]$$

$$H_r = 0$$

$$H_\theta = -\frac{E_\phi}{\eta}$$

$$H_\phi = +\frac{E_\theta}{\eta}$$



Uniform Distribution on an Infinite Ground Plane

E- and H- Far-fields

E −Plane (φ = π/2)

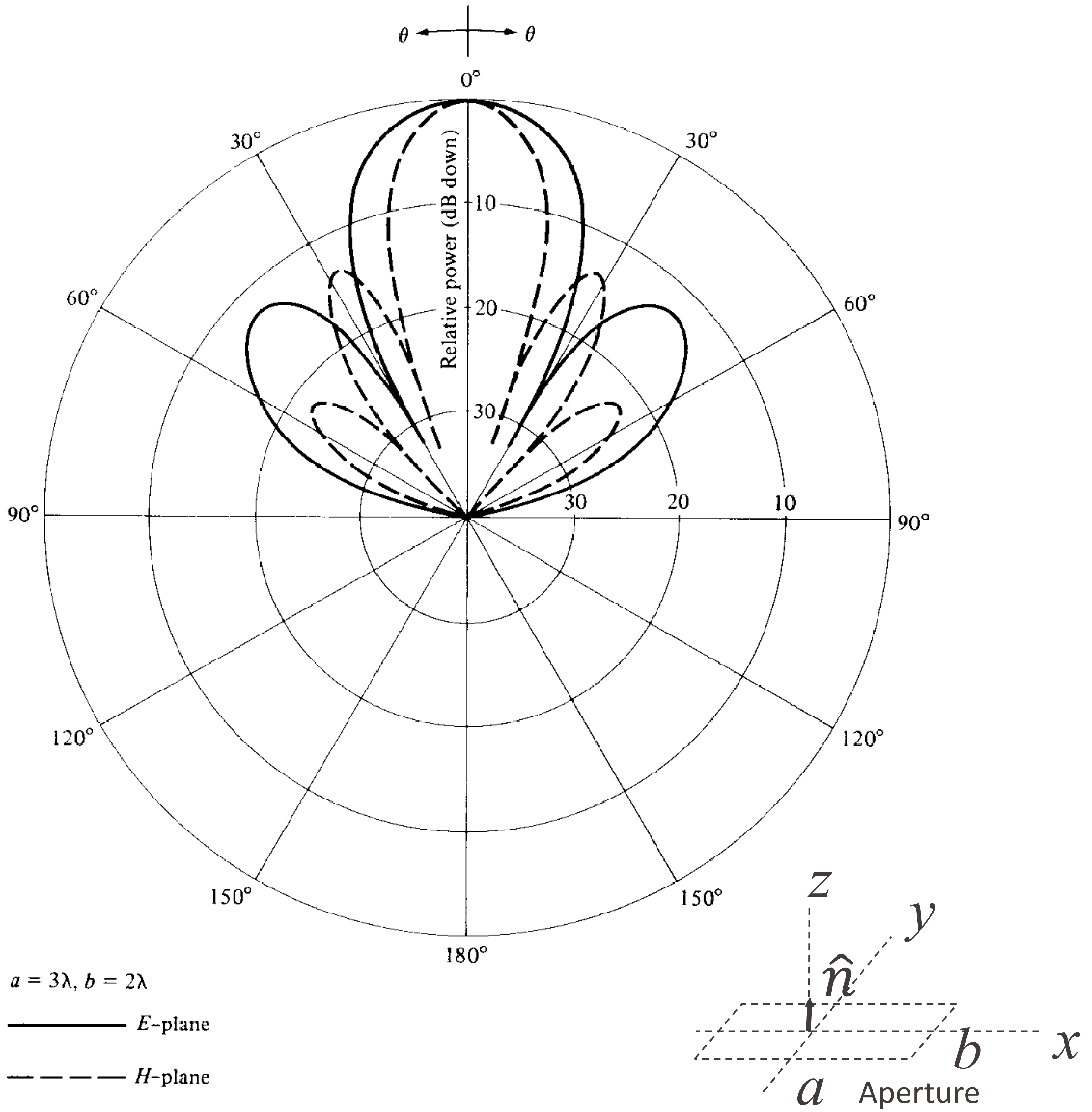
$$E_r = E_\phi = 0$$

$$E_\theta = j\frac{abkE_0e^{-jkr}}{2\pi r}\left[\frac{\sin\left(\frac{kb}{2}\sin\theta\right)}{\frac{kb}{2}\sin\theta}\right]$$

H −Plane (φ = 0)

$$E_r = E_\theta = 0$$

$$E_\phi = j\frac{abkE_0e^{-jkr}}{2\pi r}\left\{\cos\theta\left[\frac{\sin\left(\frac{ka}{2}\sin\theta\right)}{\frac{ka}{2}\sin\theta}\right]\right\}$$



E−Plane (φ = π/2) and H-plane (φ = 0) amplitude patterns for uniform distribution aperture mounted on an infinite ground plane (a = 3λ, b = 2λ).

Uniform Distribution on an Infinite Ground Plane

Since the aperture is mounted on an infinite ground plane, an alternate and much simpler method can be used to compute the radiated power. The average power density is first formed using the fields at the aperture, and it is then integrated over the physical bounds of the opening. The integration is confined to the physical bounds of the opening. The magnetic field at the aperture is given by

$$\mathbf{H}_a = -\hat{\mathbf{a}}_x \frac{E_0}{\eta}$$

where η is the intrinsic impedance, the radiated power reduces to

$$P_{\text{rad}} = \oint\!\!\oint_S \mathbf{W}_{\text{av}} \cdot d\mathbf{s} = \frac{|E_0|^2}{2\eta} \iint_{S_a} ds = ab \frac{|E_0|^2}{2\eta}$$

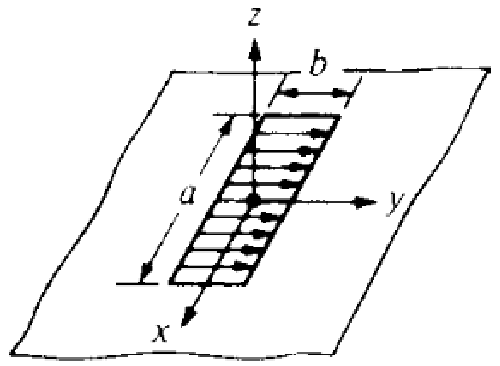
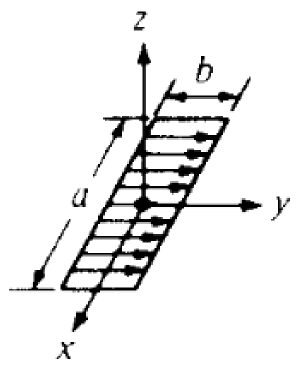
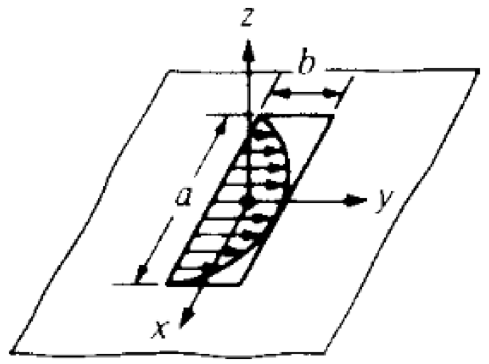
The maximum radiation intensity, U_{max}, using the fields of E_r, E_θ, *and* E_φ occurs toward $\theta = 0^\circ$ an d it is equal to

$$U_{\max} = \left(\frac{ab}{\lambda}\right)^2 \frac{|E_0|^2}{2\eta}$$

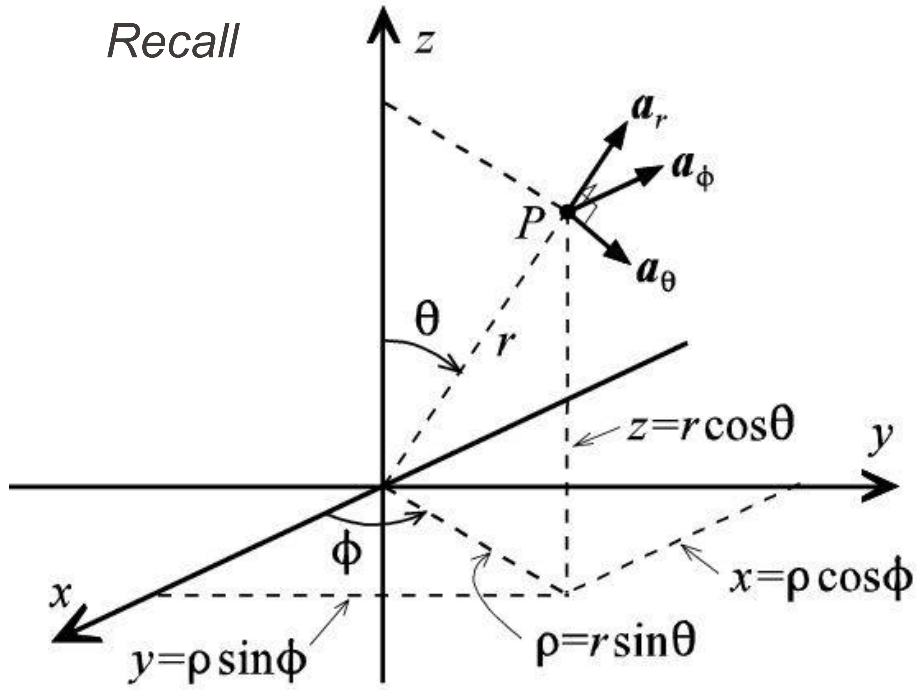
Thus the directivity is equal to

$$D_0 = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi}{\lambda^2} ab$$

Distribution Comparison on an Infinite Ground Plane

	Uniform Distribution Aperture on Ground Plane	**Uniform Distribution Aperture in Free-Space**	**TE_{10}-Mode Distribution Aperture on Ground Plane**
Aperture distribution of tangential components (analytical)	$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \left\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array} \right.$	$\left. \begin{array}{l} \mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \\ \mathbf{H}_a = -\hat{\mathbf{a}}_x \dfrac{E_0}{\eta} \end{array} \right\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}$	$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \cos\left(\dfrac{\pi}{a} x'\right) \left\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array} \right.$
Aperture distribution of tangential components (graphical)			
Equivalent	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \left\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array} \right. \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = \ 0 \quad \text{everywhere}$	$\left. \begin{array}{l} \mathbf{M}_s = -\hat{\mathbf{n}} \times \mathbf{E}_a \\ \mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}_a \end{array} \right\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array}$ $\mathbf{M}_s \simeq \mathbf{J}_s \simeq 0 \quad \text{elsewhere}$	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \left\} \begin{array}{l} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{array} \right. \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = \ 0 \quad \text{everywhere}$
Far-zone fields $X = \dfrac{ka}{2} \sin\theta \cos\phi$ $Y = \dfrac{kb}{2} \sin\theta \sin\phi$ $C = j\dfrac{abkE_0 e^{-jkr}}{2\pi r}$	$E_r = H_r = 0$ $E_\theta = C \sin\phi \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $E_\phi = C \cos\theta \cos\phi \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$	$E_r = H_r = 0$ $E_\theta = \dfrac{C}{2} \sin\phi(1+\cos\theta) \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $E_\phi = \dfrac{C}{2} \cos\phi(1+\cos\theta) \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$	$E_r = H_r = 0$ $E_\theta = -\dfrac{\pi}{2} C \sin\phi \dfrac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \dfrac{\sin Y}{Y}$ $E_\phi = -\dfrac{\pi}{2} C \cos\theta \cos\phi \dfrac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$

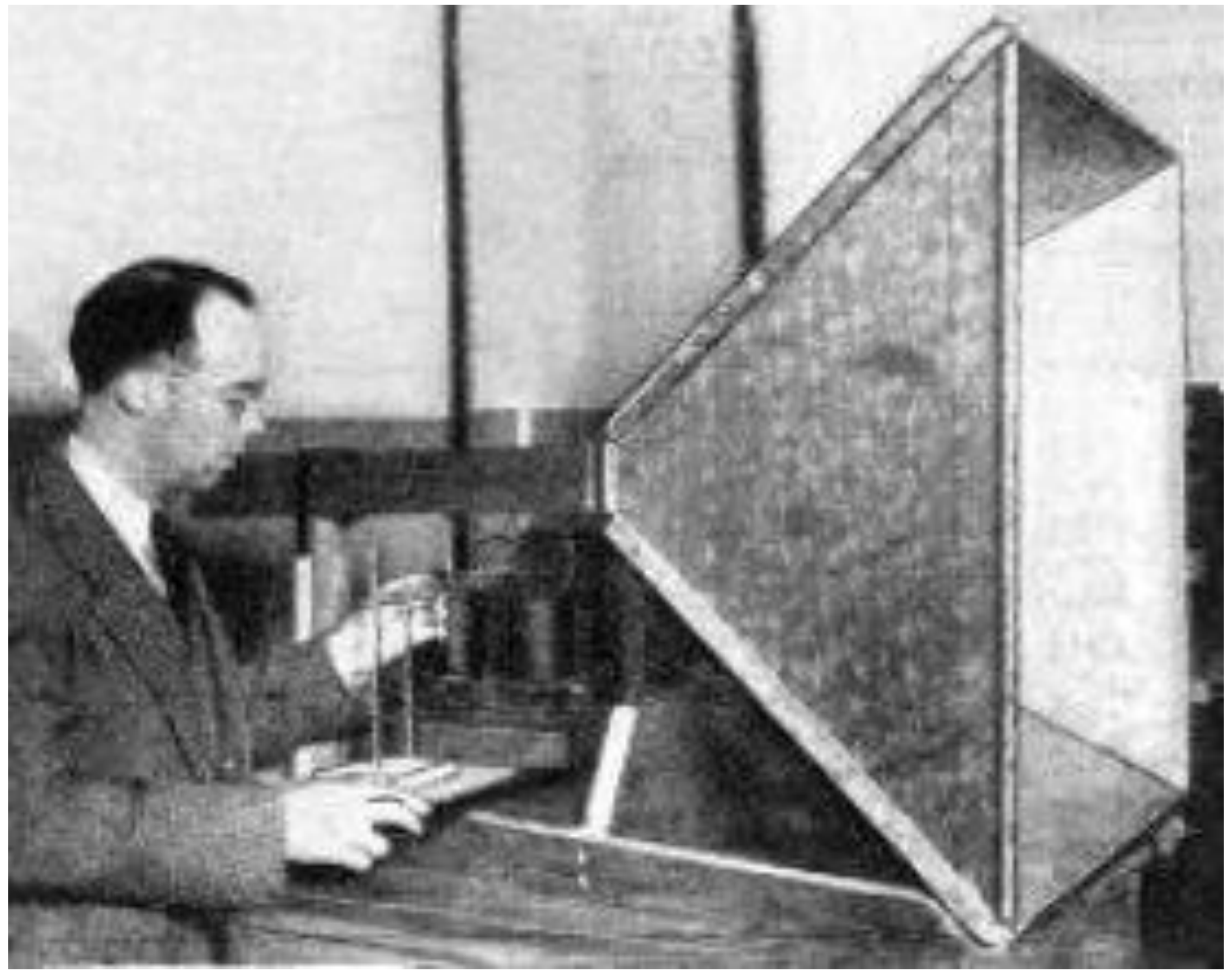
Recall



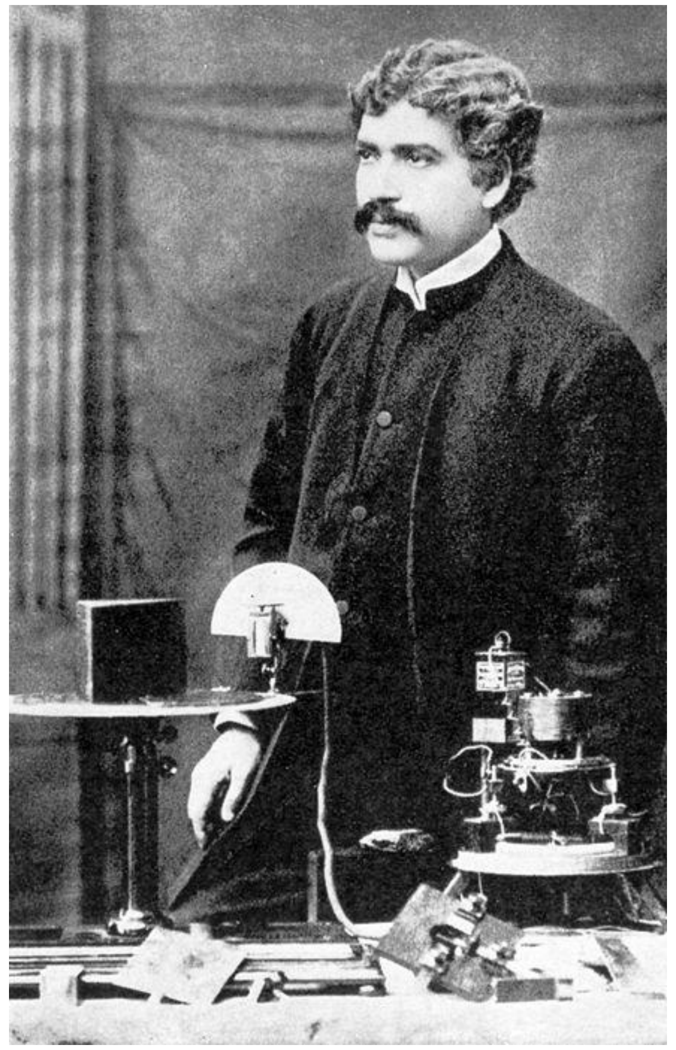
Horn Antenna

History

One of the first horn antennas was constructed in 1897 by Bengali-Indian radio researcher Jagadish Chandra Bose in his pioneering experiments with microwaves. The modern horn antenna was invented independently in 1938 by Wilmer Barrow and G. C. Southworth.



Wilmer Lanier Barrow
1903-1975



Jagadish Chandra Bose
1858-1937

Bose's 60 GHz microwave apparatus at the Bose Institute, Kolkata, India. His receiver *(left)* used a galena crystal detector inside a horn antenna and galvanometer to detect microwaves. Bose invented the crystal radio detector, waveguide, horn antenna, and other apparatus used at microwave frequencies.

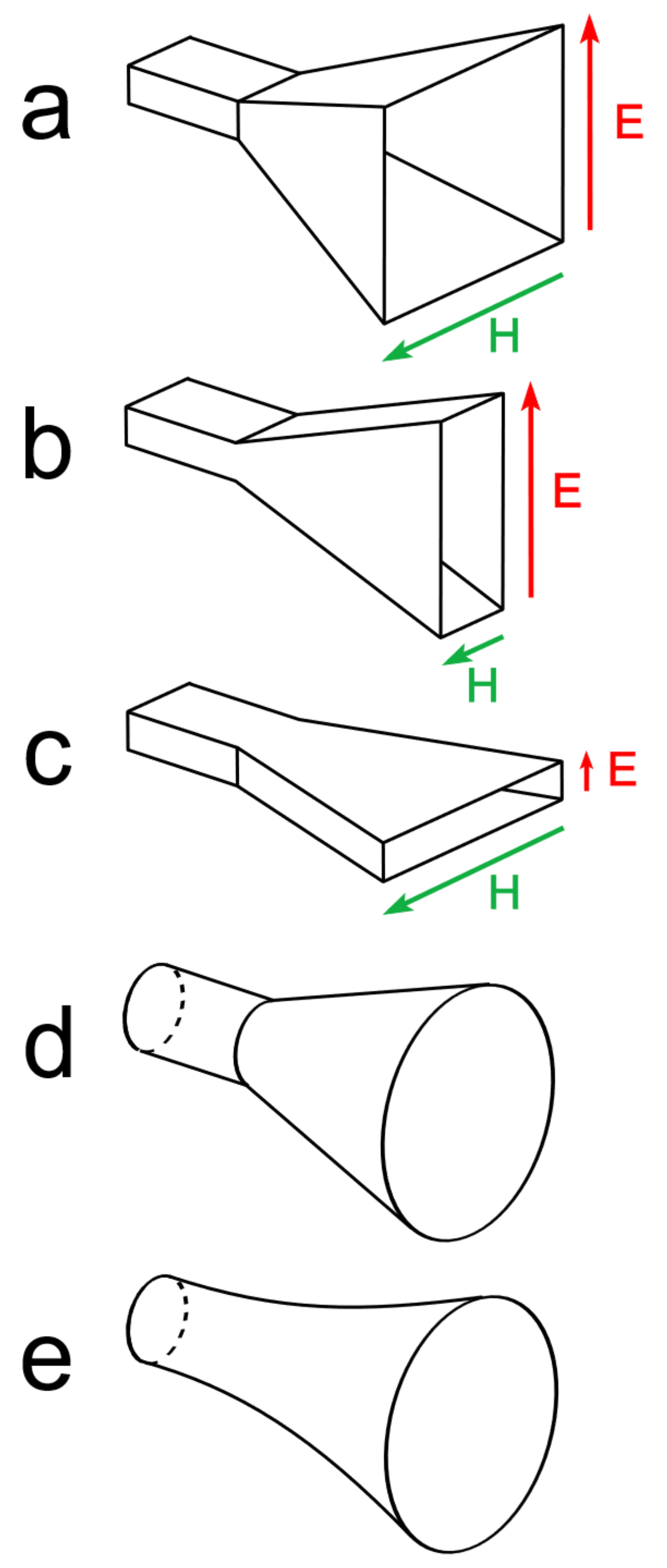
Horn Antenna

Types

The E-plane sectoral horn is one whose opening is flared in the direction of the E-field. The horn can be treated as an aperture antenna.

To develop an exact equivalent of it, it is necessary that the tangential electric and magnetic field components over a closed surface are known. The closed surface that is usually selected is an infinite plane that coincides with the aperture of the horn.

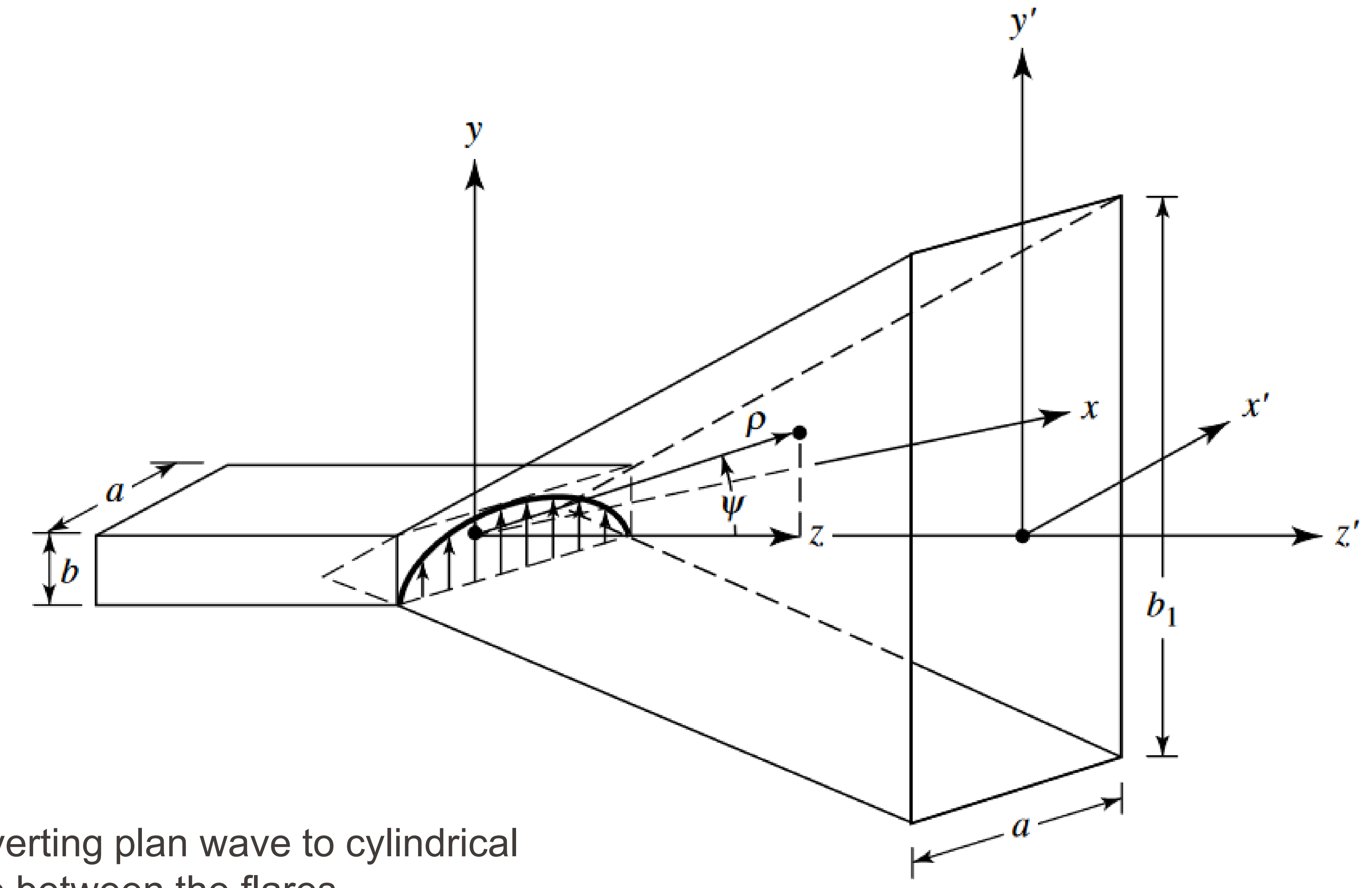
When the horn is not mounted on an infinite ground plane, the fields outside the aperture are not known and an exact equivalent cannot be formed. However, the usual approximation is to assume that the fields outside the aperture are zero.



Typical electromagnetic horn antenna configurations:
(a) Pyramidal horn
(b) E-plane sectoral horn
(c) H-plane sectoral horn
(d) Conical horn
(e) Exponential horn

Horn Antenna

E-Plane Sectoral Horn



It can be shown that if the (1) fields of the feed waveguide are those of its dominant TE_{10} mode and (2) horn length is large compared to the aperture dimensions, the lowest order mode fields at the aperture of the hornare given by

$$E'_z = E'_x = H'_y = 0$$

TE_{10}

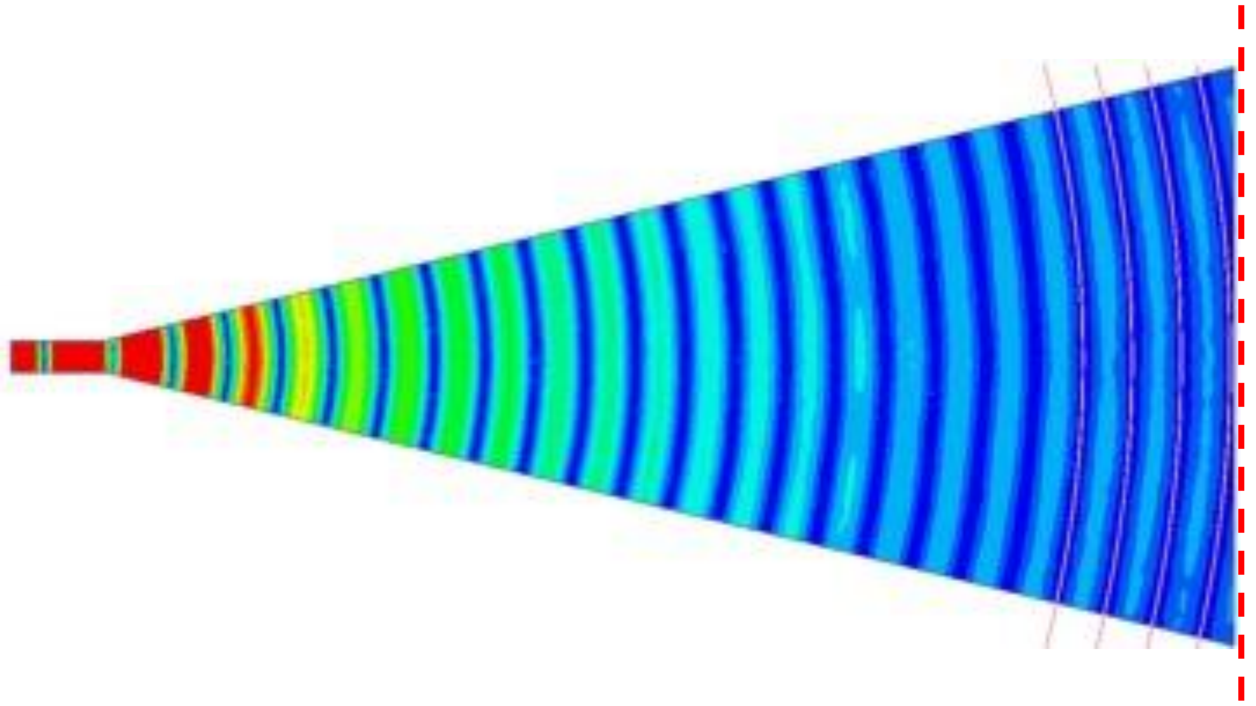
$$E'_y(x', y') \simeq E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

Converting plan wave to cylindrical wave between the flares

$$H'_z(x', y') \simeq jE_1\left(\frac{\pi}{ka\eta}\right)\sin\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$H'_x(x', y') \simeq -\frac{E_1}{\eta}\cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$\rho_1 = \rho_e \cos\psi_e$$



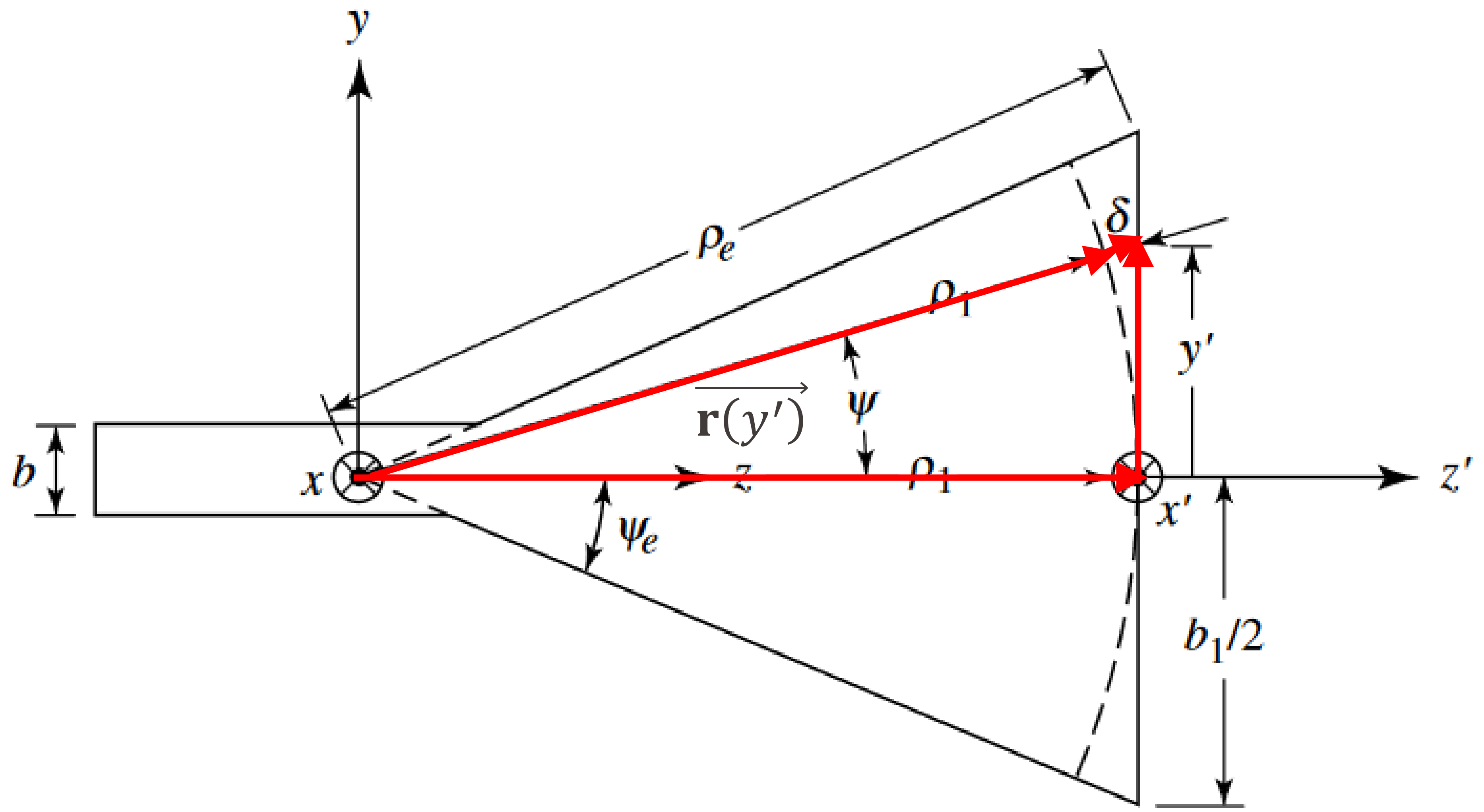
where E_1 is a constant. The primes are used to indicate the fields at the aperture of the horn. The expressions are similar to the fields of a TE_{10} -mode for a rectangular waveguide with aperture dimensions of ***a*** and ***b₁*** (***b₁*** > ***a***). The only difference is the complex exponential term which is used here to represent the quadratic phase variations of the fields over the aperture of the horn.

Horn Antenna

Phase deviation

Let the throat (phase center) of the horn be the origin, and the aperture lie in the $x - y$ plane.
The on-axis ray (to the aperture center) is represented by

$$\mathbf{r}_0 = \rho_1 \,\hat{\mathbf{z}},$$



where $\rho_1$is the axial path length.
A ray reaching a point on the aperture displaced by $y'$from the axis is

$$|\mathbf{r}(y')| = |y' \,\hat{\mathbf{y}} + \rho_1 \,\hat{\mathbf{z}}| = \rho_1 + \delta(y').$$

Its total path length is the magnitude of this vector:

$$|\mathbf{r}(y')| = \sqrt{\rho_1^2 + y'^2}. \qquad \Rightarrow \quad [\rho_1 + \delta(y')]^2 = \rho_1^2 + y'^2.$$

For small deviation, $\delta \ll \rho_1$, expand to get

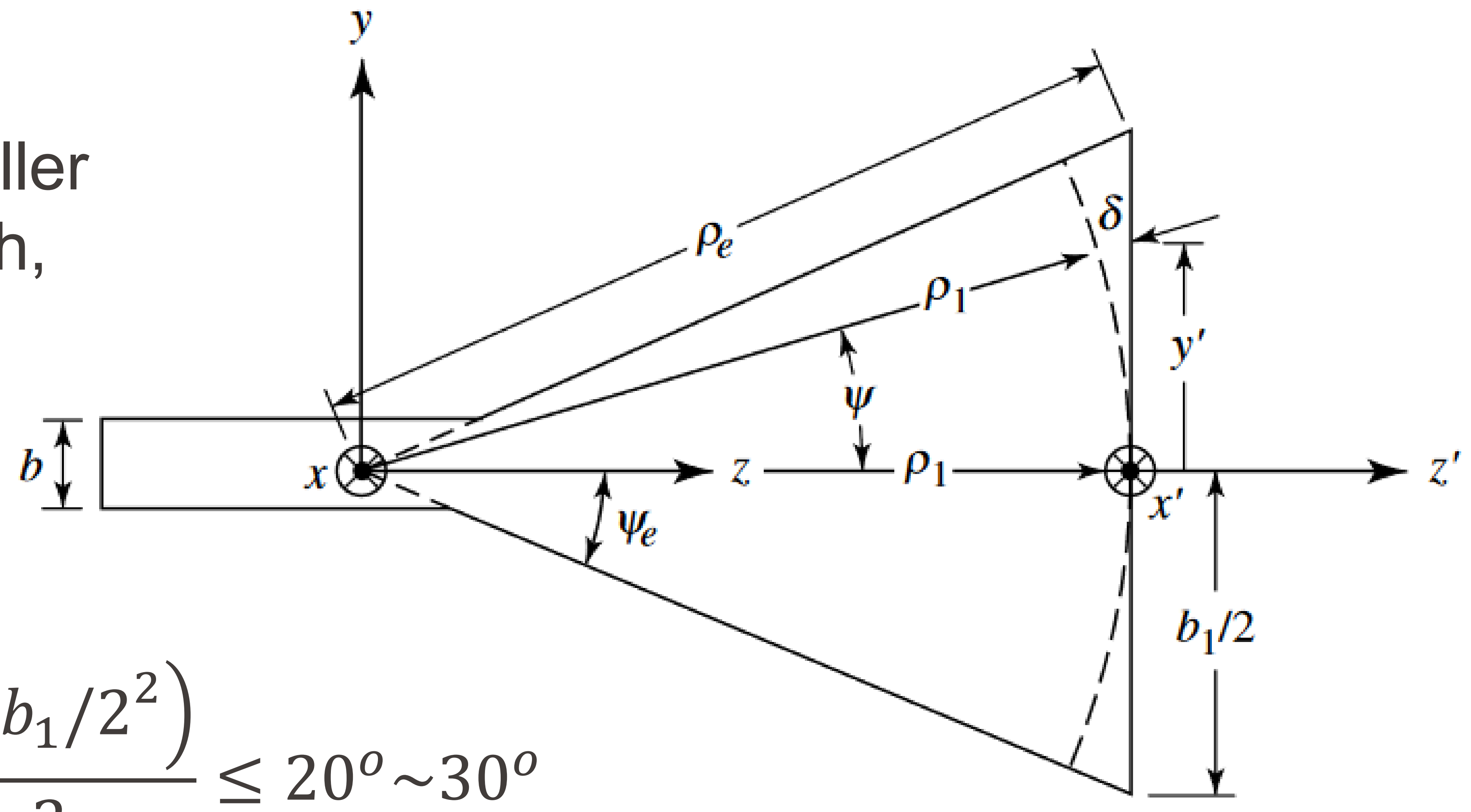
$$\delta(y') = \sqrt{\rho_1^2 + y'^2} - \rho_1 \approx \frac{y'^2}{2\rho_1}.$$

Horn Antenna

Maximum phase deviation

The *maximum phase* deviation at the aperture of a horn is the largest difference in phase between the center and the edge of the horn's aperture, indicating the non-planarity of the radiating wavefront.

It determines how uniformly the aperture radiates — smaller phase deviation leads to higher gain, narrower beamwidth, and better radiation efficiency.



Phase error at the aperture is then

$$\Delta\phi(y') = \frac{2\pi}{\lambda}\,\delta(y')\Big|_{y'=b_1/2} \Rightarrow \Delta\phi_{\max} = \frac{2\pi}{\lambda}\,\frac{\left(b_1/2^2\right)}{2\rho_1} \leq 20^o\sim30^o$$

For acceptable radiation performance

The total flare angle of the horn

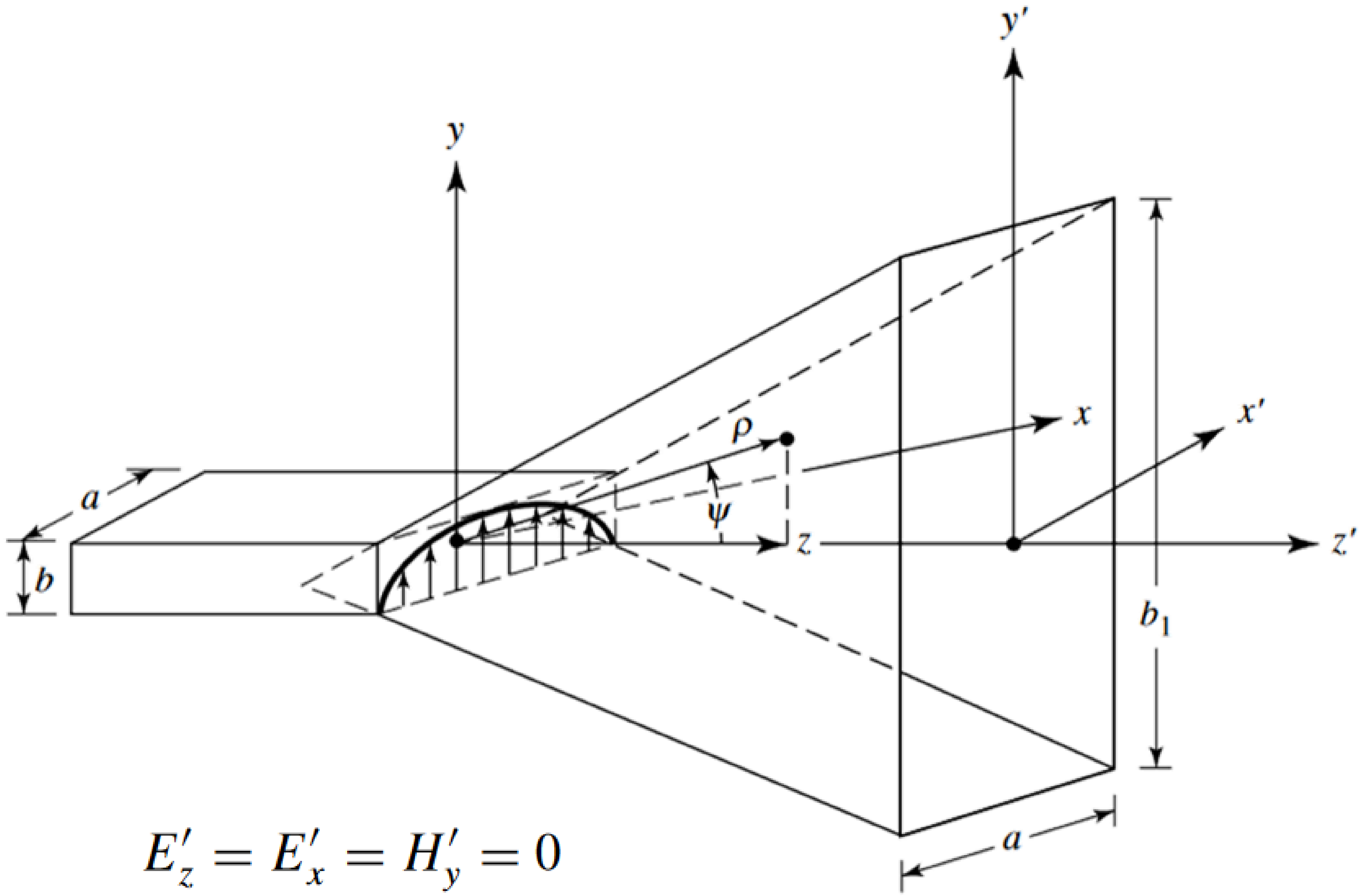
$$2\psi_e = 2tan^{-1}\left(\frac{\frac{b_1}{2}}{\rho_1}\right) = 2tan^{-1}\left(\frac{b_1}{2\rho_1}\right)$$

Antennas

Horn Antenna

N- and L-functions

To find the fields radiated by the horn, only the *tangential components* of the E- and/or H-fields over a closed surface must be known. The closed surface is chosen to coincide with an infinite plane passing through the mouth of the horn.



$$E'_z = E'_x = H'_y = 0$$

$$E'_y(x', y') \simeq E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$H'_z(x', y') \simeq jE_1\left(\frac{\pi}{ka\eta}\right)\sin\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$H'_x(x', y') \simeq -\frac{E_1}{\eta}\cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$\rho_1 = \rho_e \cos\psi_e$$

→

$$E'_z = E'_x = H'_y = 0$$

$$E'_y(x', y') \simeq E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$H'_x(x', y') \simeq -\frac{E_1}{\eta}\cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$\rho_1 = \rho_e \cos\psi_e$$

Horn Antenna

N- and L-functions

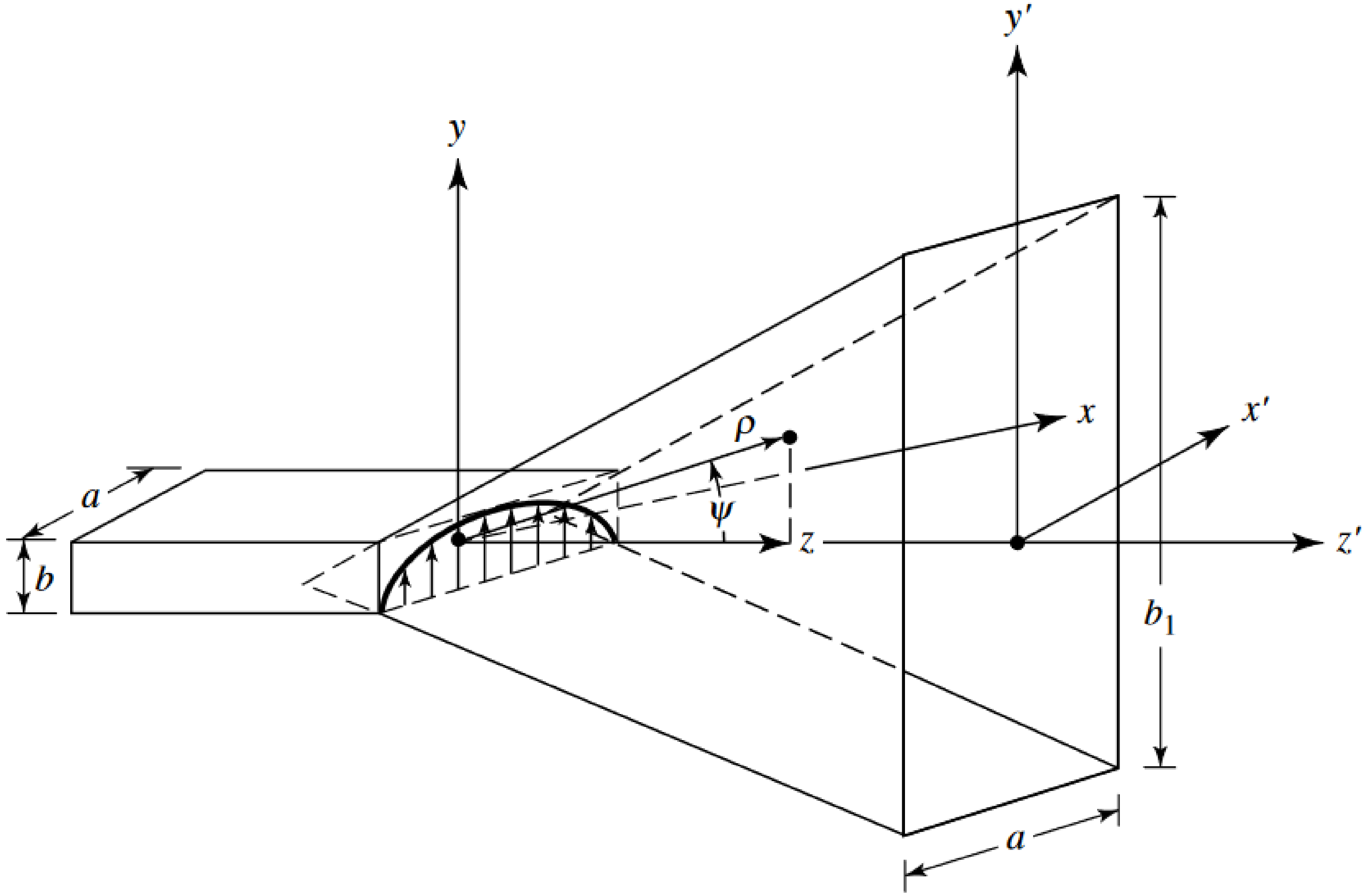
Since we have both E_y and H_x over the surface

$$E'_z = E'_x = H'_y = 0$$

$$E'_y(x', y') \simeq E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$H'_x(x', y') \simeq -\frac{E_1}{\eta} \cos\left(\frac{\pi}{a}x'\right) e^{-j[ky'^2/(2\rho_1)]}$$

$$\rho_1 = \rho_e \cos\psi_e$$



Thus

$$J_s = \hat{n} \times H_a = \hat{z} \times H_x = \boldsymbol{J_y}$$

$$M_s = -\hat{n} \times E_a = -\hat{z} \times E_y = \boldsymbol{M_x}$$

$$\left.\begin{aligned} J_y &= -\frac{E_1}{\eta} \cos\left(\frac{\pi}{a}x'\right) e^{-jk\delta(y')} \\ M_x &= E_1 \cos\left(\frac{\pi}{a}x'\right) e^{-jk\delta(y')} \end{aligned}\right\} \begin{aligned} -a/2 \le x' \le a/2 \\ -b_1/2 \le y' \le b_1/2 \end{aligned}$$

$$\mathbf{J}_s = \mathbf{M}_s = 0 \quad \text{elsewhere}$$

Recall:

$\hat{x} \rightarrow \hat{y} \rightarrow \hat{z} \rightarrow \hat{x}$

Horn Antenna

N- and L-functions

Since we have

$$J_s = \hat{n} \times H_a = \hat{z} \times H_x = \boldsymbol{J_y}$$

$$M_s = -\hat{n} \times E_a = -\hat{z} \times E_y = \boldsymbol{M_x}$$

Thus

$$N_\theta = \iint_S [J_x \cos\theta \cos\phi + J_y \cos\theta \sin\phi - J_z \sin\theta] e^{+jkr' \cos\psi}\, ds'$$

$$N_\phi = \iint_S [-J_x \sin\phi + J_y \cos\phi] e^{+jkr' \cos\psi}\, ds'$$

$$L_\theta = \iint_S [M_x \cos\theta \cos\phi + M_y \cos\theta \sin\phi - M_z \sin\theta] e^{+jkr' \cos\psi}\, ds'$$

$$L_\phi = \iint_S [-M_x \sin\phi + M_y \cos\phi] e^{+jkr' \cos\psi}\, ds'$$

Antennas

Horn Antenna

N- and L-functions

Starting with N_θ

$$N_\theta = \iint_S [J_x \cos\theta \cos\phi + J_y \cos\theta \sin\phi - J_z \sin\theta] e^{+jkr'\cos\psi} \, ds'$$

Thus

$$N_\theta = -\frac{E_1}{\eta} \cos\theta \sin\phi I_1 I_2$$

where

$$I_1 = \int_{-a/2}^{+a/2} \cos\left(\frac{\pi}{a}x'\right) e^{jkx'\sin\theta\cos\phi} \, dx' = -\left(\frac{\pi a}{2}\right) \left[\frac{\cos\left(\frac{ka}{2}\sin\theta\cos\phi\right)}{\left(\frac{ka}{2}\sin\theta\cos\phi\right)^2 - \left(\frac{\pi}{2}\right)^2} \right]$$

$$I_2 = \int_{-b_1/2}^{+b_1/2} e^{-jk[\delta(y') - y'\sin\theta\sin\phi]} \, dy'$$

Horn Antenna

N- and L-functions

The integral can also be evaluated in terms of cosine and sine Fresnel integrals.

$$I_2 = \sqrt{\frac{\pi \rho_1}{k}} e^{j(k_y^2 \rho_1/2k)} \{[C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]\}$$

where

$$t_1 = \sqrt{\frac{1}{\pi k \rho_1}} \left(-\frac{k b_1}{2} - k_y \rho_1 \right)$$

$$t_2 = \sqrt{\frac{1}{\pi k \rho_1}} \left(\frac{k b_1}{2} - k_y \rho_1 \right)$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2} t^2\right) dt$$

Horn Antenna

N- and L-functions

Thus it can be written as

$$N_\theta = E_1 \frac{\pi a}{2}\sqrt{\frac{\pi \rho_1}{k}} e^{j(k_y^2 \rho_1/2k)} \times \left\{ \frac{\cos\theta \sin\phi}{\eta} \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$

where

$$\boxed{k_x = k \sin\theta \cos\phi}$$

$$\boxed{k_y = k \sin\theta \sin\phi}$$

$$\boxed{F(t_1, t_2) = [C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]}$$

Horn Antenna

N- and L-functions

In a similar manner, N_ϕ , L_θ , L_ϕ are

$$N_\phi = E_1 \frac{\pi a}{2} \sqrt{\frac{\pi \rho_1}{k}} e^{j(k_y^2 \rho_1/2k)} \left\{ \frac{\cos\phi}{\eta} \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$

$$L_\theta = E_1 \frac{\pi a}{2} \sqrt{\frac{\pi \rho_1}{k}} e^{j(k_y^2 \rho_1/2k)} \times \left\{ -\cos\theta \cos\phi \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$

$$L_\phi = E_1 \frac{\pi a}{2} \sqrt{\frac{\pi \rho_1}{k}} e^{j(k_y^2 \rho_1/2k)} \left\{ \sin\phi \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$

Horn Antenna

N- and L-functions

Having established the N_θ , N_ϕ , L_θ , L_ϕ are

$$E_r \simeq 0$$

$$E_\theta \simeq -\frac{jke^{-jkr}}{4\pi r}(L_\phi + \eta N_\theta)$$

$$E_\phi \simeq +\frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\phi)$$

$$H_r \simeq 0$$

$$H_\theta \simeq \frac{jke^{-jkr}}{4\pi r}\left(N_\phi - \frac{L_\theta}{\eta}\right)$$

$$H_\phi \simeq -\frac{jke^{-jkr}}{4\pi r}\left(N_\theta + \frac{L_\phi}{\eta}\right)$$

Horn Antenna

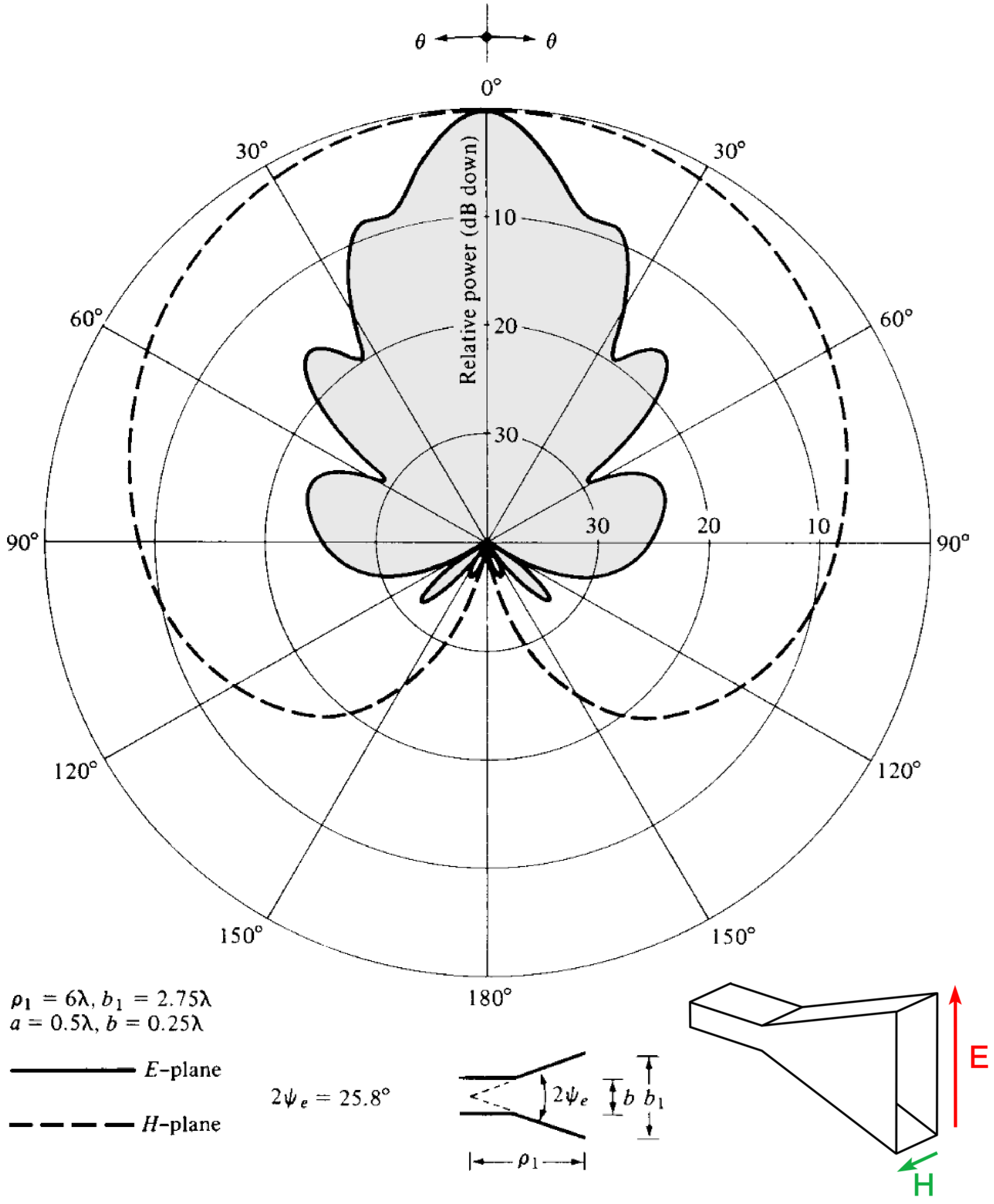
E- and H- Far-fields

The electric field components radiated by the horn can be obtained by

$$E_r = 0$$

$$E_\theta = -j\frac{a\sqrt{\pi k \rho_1} E_1 e^{-jkr}}{8r} \times \left\{ e^{j(k_y^2\rho_1/2k)} \sin\phi(1+\cos\theta) \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$

$$E_\phi = -j\frac{a\sqrt{\pi k \rho_1} E_1 e^{-jkr}}{8r} \times \left\{ e^{j(k_y^2\rho_1/2k)} \cos\phi(\cos\theta+1) \left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2} \right] F(t_1, t_2) \right\}$$



E-Plane ($\phi = \pi/2$)

$$E_r = E_\phi = 0$$

H-Plane ($\phi = 0$)

$$E_r = E_\theta = 0$$

E- and H -plane patterns of an ***E-plane sectoral horn***.

Antennas

Horn Antenna

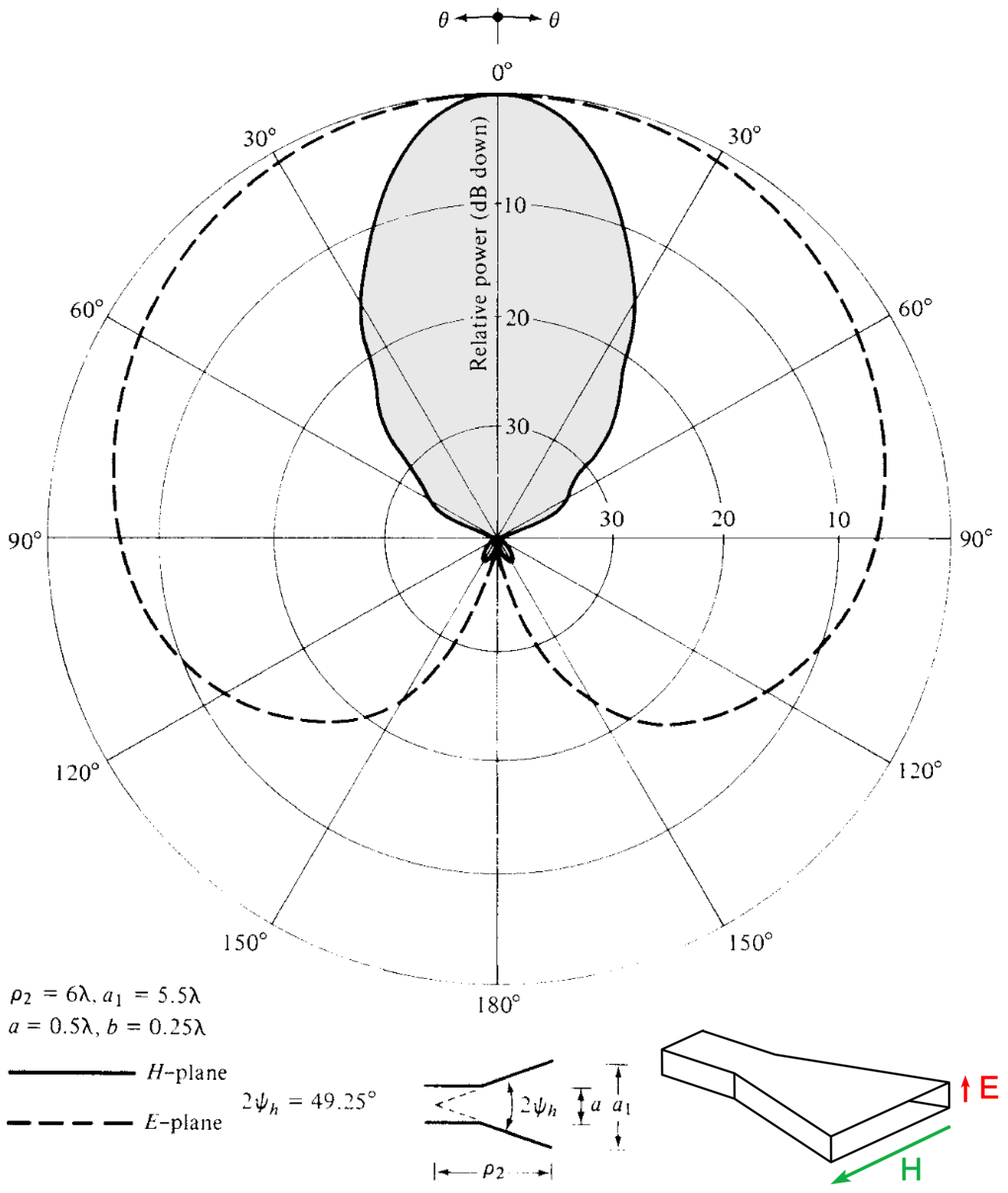
E- and H- Far-fields

The electric field components radiated by the horn can be obtained by

$$E_r = 0$$

$$E_\theta = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r} \times \left\{ \sin\phi(1+\cos\theta)\frac{\sin Y}{Y}\left[e^{jf_1}F(t_1',t_2') + e^{jf_2}F(t_1'',t_2'')\right] \right\}$$

$$E_\phi = jE_2 \frac{b}{8}\sqrt{\frac{k\rho_2}{\pi}}\frac{e^{-jkr}}{r} \times \left\{ \cos\phi(\cos\theta+1)\frac{\sin Y}{Y}\left[e^{jf_1}F(t_1',t_2') + e^{jf_2}F(t_1'',t_2'')\right] \right\}$$



E- and H -plane patterns of an ***H-plane sectoral horn***.

Horn Antenna

Directivity

The directivity is one of the parameters that is often used as a figure of merit to describe the performance of an antenna. To find the directivity, the maximum radiation is formed. That is,

$$U_{\max} = U(\theta, \phi)|_{\max} = \frac{r^2}{2\eta}|\mathbf{E}|^2_{\max}$$

For most horn antennas $|\mathbf{E}|_{max} = |\mathbf{E}_\theta + \mathbf{E}_\phi|_{max}$ is directed nearly along the z-axis ($\theta = 0^\circ$). Thus,

$$|\mathbf{E}|_{\max} = \sqrt{|E_\theta|^2_{\max} + |E_\phi|^2_{\max}} = \frac{2a\sqrt{\pi k \rho_1}}{\pi^2 r}|E_1||F(t)|$$

where

$$|E_\theta|_{\max} = \frac{2a\sqrt{\pi k \rho_1}}{\pi^2 r}|E_1 \sin\phi F(t)|$$

$$|E_\phi|_{\max} = \frac{2a\sqrt{\pi k \rho_1}}{\pi^2 r}|E_1 \cos\phi F(t)|$$

$$F(t) = [C(t) - jS(t)]$$

$$t = \frac{b_1}{2}\sqrt{\frac{k}{\pi \rho_1}} = \frac{b_1}{\sqrt{2\lambda\rho_1}}$$

Horn Antenna

Directivity

Thus

$$U_{\max} = \frac{r^2}{2\eta}|\mathbf{E}|^2_{\max} = \frac{2a^2 k\rho_1}{\eta\pi^3}|E_1|^2|F(t)|^2$$
$$= \frac{4a^2\rho_1|E_1|^2}{\eta\lambda\pi^2}|F(t)|^2$$

where

$$|F(t)|^2 = \left[C^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right) + S^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right)\right]$$

Horn Antenna

Directivity

The total power radiated can be found by simply integrating the average power density over the aperture of the horn.

$$P_{\text{rad}} = \frac{1}{2}\iint_{S_0} \text{Re}(\mathbf{E}' \times \mathbf{H}'^{*}) \cdot d\mathbf{s} = \frac{1}{2\eta}\int_{-b_1/2}^{+b_1/2}\int_{-a/2}^{+a/2} |E_1|^2 \cos^2\left(\frac{\pi}{a}x'\right) dx'\, dy'$$

Thus

$$P_{\text{rad}} = |E_1|^2 \frac{b_1 a}{4\eta}$$

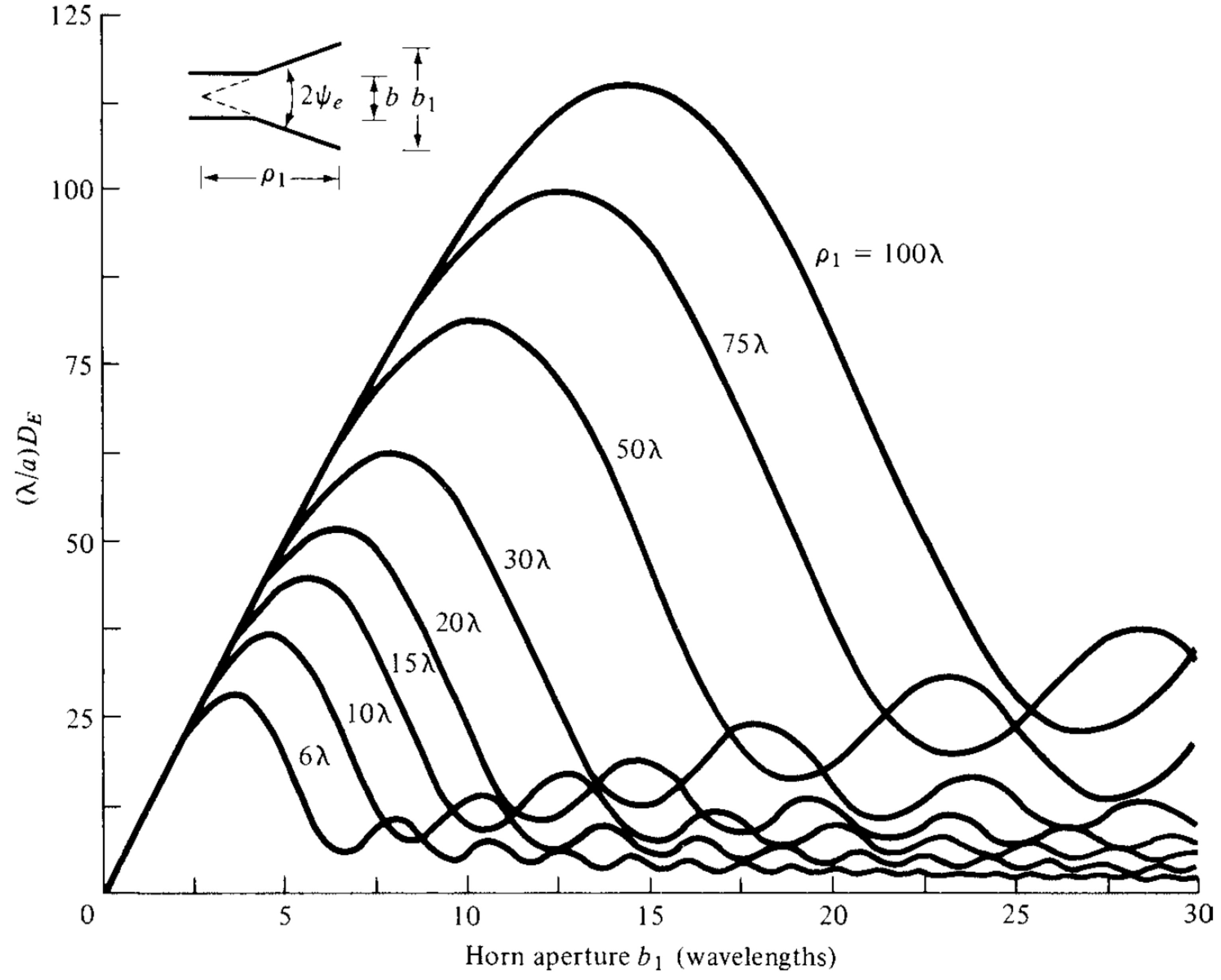
and

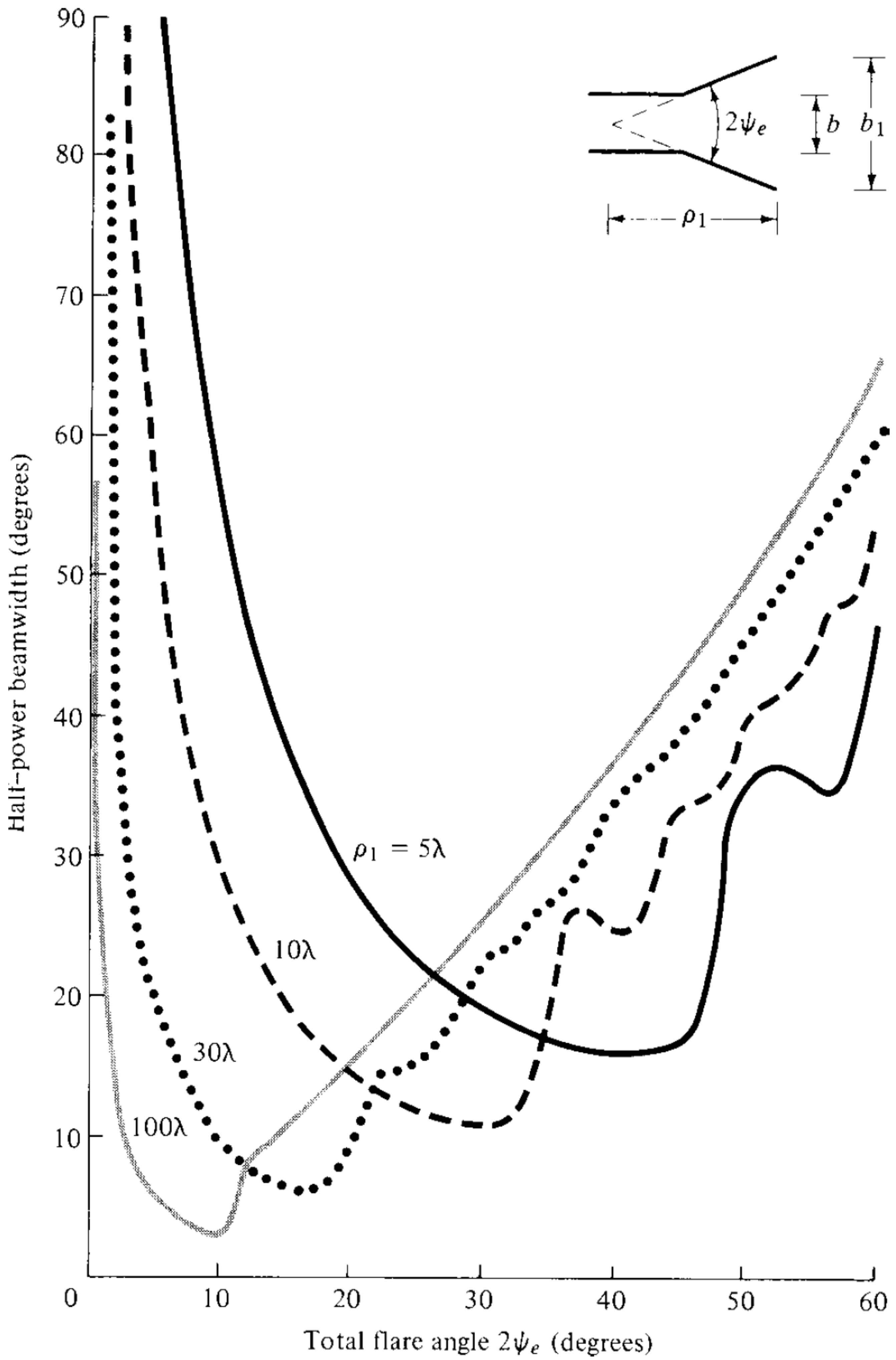
$$D_E = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{64 a\rho_1}{\pi\lambda b_1}|F(t)|^2$$
$$= \frac{64 a\rho_1}{\pi\lambda b_1}\left[C^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right) + S^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right)\right]$$

Horn Antenna

Directivity

For a given length, the horn exhibits a monotonic decrease in half-power beamwidth and an increase in directivity up to a certain flare. Beyond that point a monotonic increase in beamwidth and decrease in directivity is indicated followed by rises and falls. The increase in beamwidth and decrease in directivity beyond a certain flare indicate the broadening of the main beam.





Horn Antenna

Pyramidal Horn

To simplify the analysis and to maintain a modeling that leads to computations that have been shown to correlate well with experimental data, the tangential components of the E- and H -fields over the aperture of the hornare approximated by

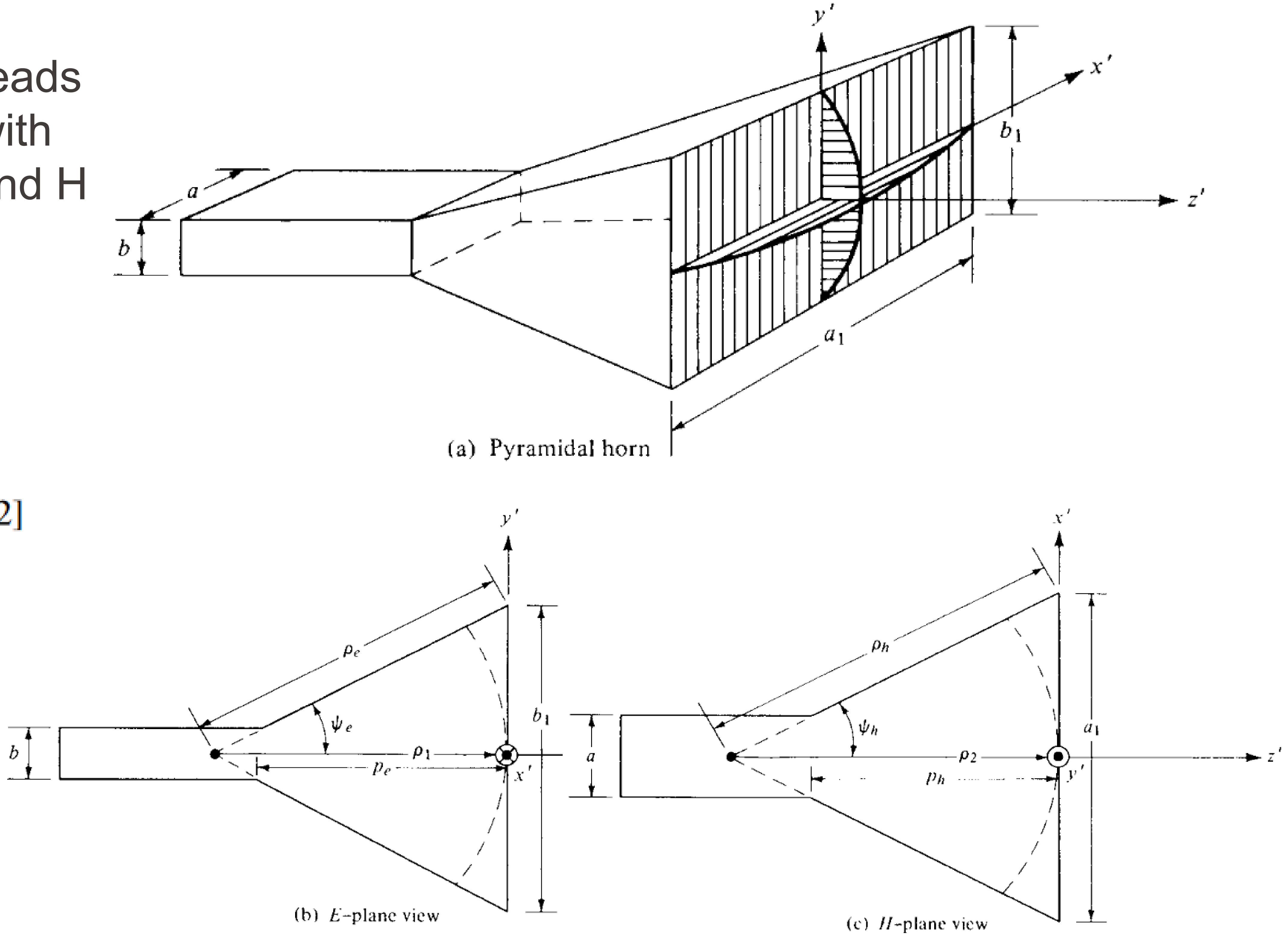
$$E'_y(x', y') = E_0 \cos\left(\frac{\pi}{a_1}x'\right) e^{-j[k(x'^2/\rho_2 + y'^2/\rho_1)/2]}$$

$$H'_x(x', y') = -\frac{E_0}{\eta} \cos\left(\frac{\pi}{a_1}x'\right) e^{-j[k(x'^2/\rho_2 + y'^2/\rho_1)/2]}$$

and the equivalent current densities by

$$J_y(x', y') = -\frac{E_0}{\eta} \cos\left(\frac{\pi}{a_1}x'\right) e^{-j[k(x'^2/\rho_2 + y'^2/\rho_1)/2]}$$

$$M_x(x', y') = E_0 \cos\left(\frac{\pi}{a_1}x'\right) e^{-j[k(x'^2/\rho_2 + y'^2/\rho_1/2]}$$



(a) Pyramidal horn

(b) E-plane view

(c) H-plane view

$$p_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

$$p_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

Horn Antenna

Pyramidal Horn

The far-zone E- and H-field components reduce to

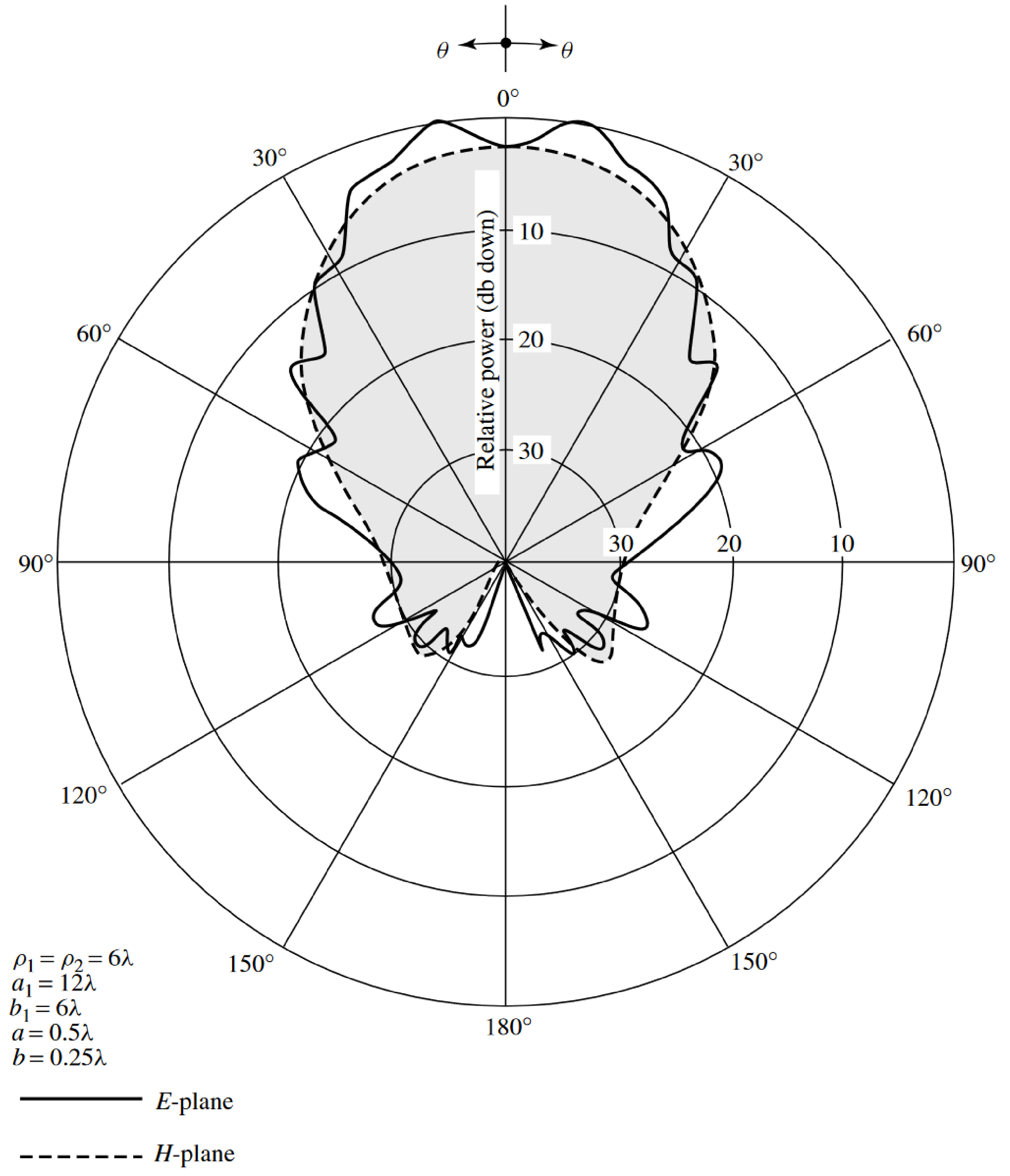
$$E_r = 0$$

$$E_\theta = -j\frac{ke^{jkr}}{4\pi r}[L_\phi + \eta N_\theta]$$

$$= j\frac{kE_0 e^{-jkr}}{4\pi r}[\sin\phi(1+\cos\theta)I_1 I_2]$$

$$E_\phi = +j\frac{ke^{-jkr}}{4\pi r}[L_\theta - \eta N_\phi]$$

$$= j\frac{kE_0 e^{-jkr}}{4\pi r}[\cos\phi(\cos\theta+1)I_1 I_2]$$



E- and H -plane patterns of an ***Pyramidal Horn***.

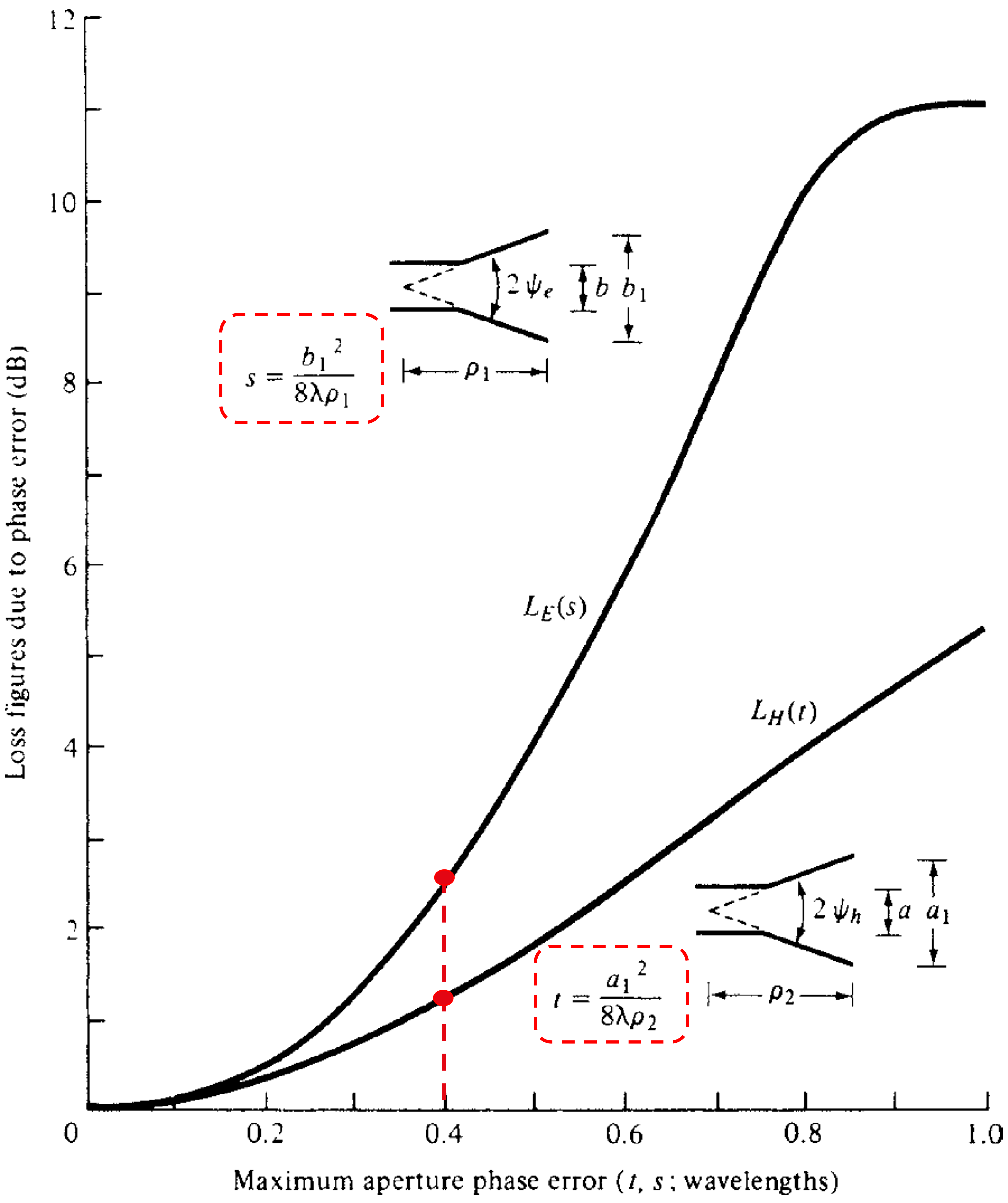
Horn Antenna

Directivity

The directivity (in dB) of a pyramidal horn, over isotropic, can also be approximated by

$$D_p(\text{dB}) = 10\left[1.008 + \log_{10}\left(\frac{a_1 b_1}{\lambda^2}\right)\right] - (L_e + L_h)$$

where L_e and L_h represent, respectively, the losses (*in dB*) due to phase errors in the *E*- and *H*-planes of the horn which are found plotted in the figure.



Antennas

Horn Antenna

Pyramidal Horn: Design Procedure

The pyramidal horn is widely used as a standard to make gain measurements of other antennas, and as such it is often referred to as a standard gain horn. To design a pyramidal horn, one usually knows the desired gain G_0 and the dimensions a, b of the rectangular feed waveguide. The objective of the design is to determine the remaining dimensions (a_1, b_1, ρ_e, ρ_h, P_e, an d_{Ph}) that will lead to an optimum gain. The procedure that follows can be used to accomplish this,

1. As a first step of the design, find the value of χ

$$\chi\,(\text{trial}) = \chi_1 = \frac{G_0}{2\pi\sqrt{2\pi}}$$

2. Once the correct χ has been found, determine ρ_e and ρ_h using

$$\frac{\rho_e}{\lambda} = \chi \qquad \frac{\rho_h}{\lambda} = \frac{G_0^2}{8\pi^3}\left(\frac{1}{\chi}\right)$$

3. Find the corresponding values of a_1 and b_1 using

$$a_1 = \sqrt{3\lambda\rho_2} \simeq \sqrt{3\lambda\rho_h} = \frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi\chi}}\lambda$$

$$b_1 = \sqrt{2\lambda\rho_1} \simeq \sqrt{2\lambda\rho_e} = \sqrt{2\chi\lambda}$$

→

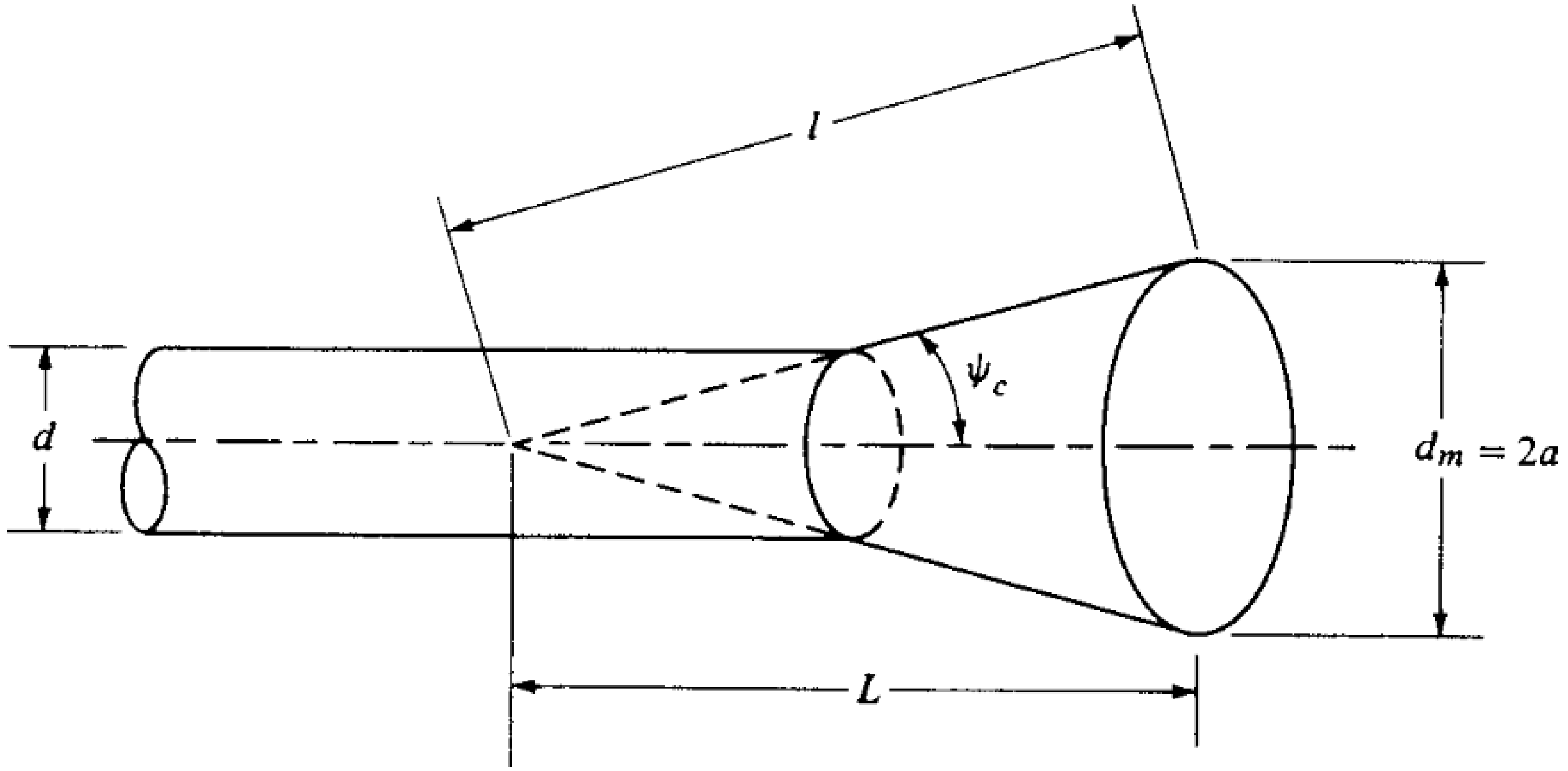
$$p_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

$$p_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

Horn Antenna

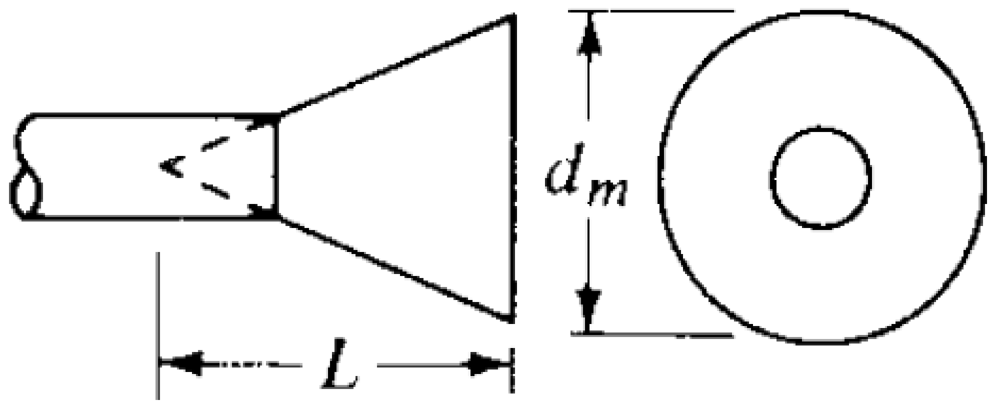
Conical Horn

Another very practical microwave antenna is the conical horn. While the pyramidal, E-, and H -plane sectoral horns are usually fed by a rectangular waveguide, the feed of a conical horn is often a circular waveguide.



The directivity (in dB) of a conical horn, with an aperture efficiency of Iap and aperture circumference C, can be computed using

$$D_c(\text{dB}) = 10\log_{10}\left[\epsilon_{ap}\frac{4\pi}{\lambda^2}(\pi a^2)\right] = 10\log_{10}\left(\frac{C}{\lambda}\right)^2 - L(s)$$



Schematic of a ***Conical Horn***.

where ***a*** is the radius of the horn at the aperture. The first term represents the directivity of a uniform circular aperture whereas the second term, is a correction figure to account for the loss indirectivity due to the aperture efficiency. Usually the term is referred to as ***loss figure*** which can be computed (in decibels) using

$$L(s) \simeq (0.8 - 1.71s + 26.25s^2 - 17.79s^3) \qquad \text{and} \qquad s = \frac{d_m^2}{8\lambda l}$$

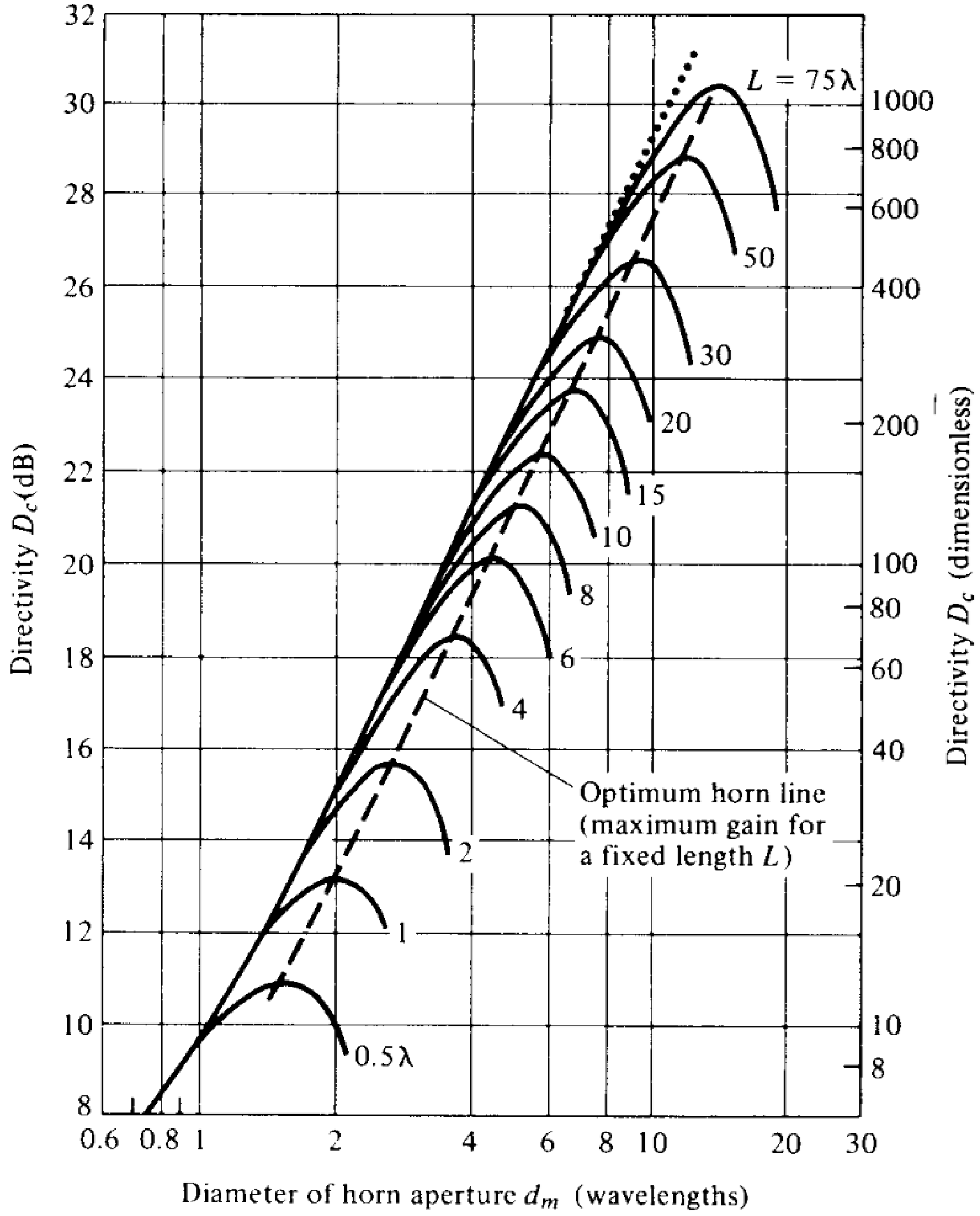
Horn Antenna

Conical Horn

The directivity of a conical horn is optimum when its diameter is equal to

$$d_m \simeq \sqrt{3l\lambda}$$

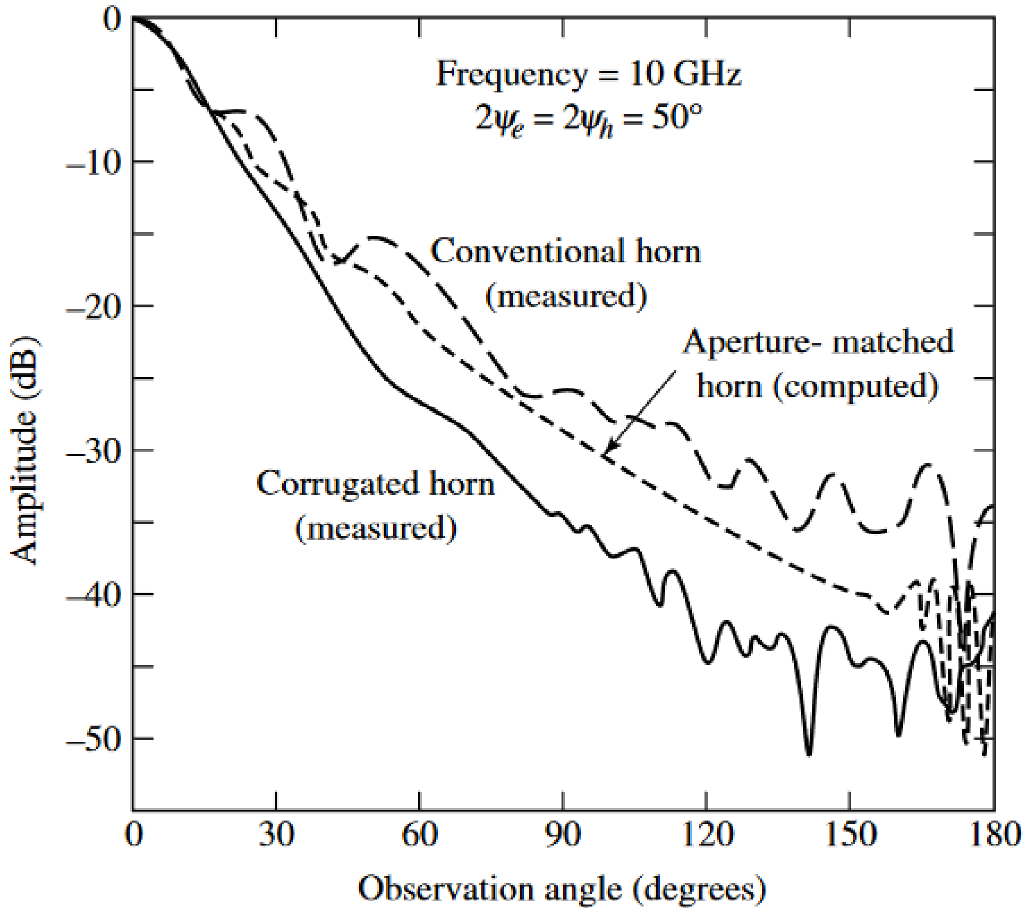
which corresponds to a maximum aperture phase deviation of s = 3/8 (wavelengths) and a loss figure of about 2.9 dB (or an aperture efficiency of about 51%).



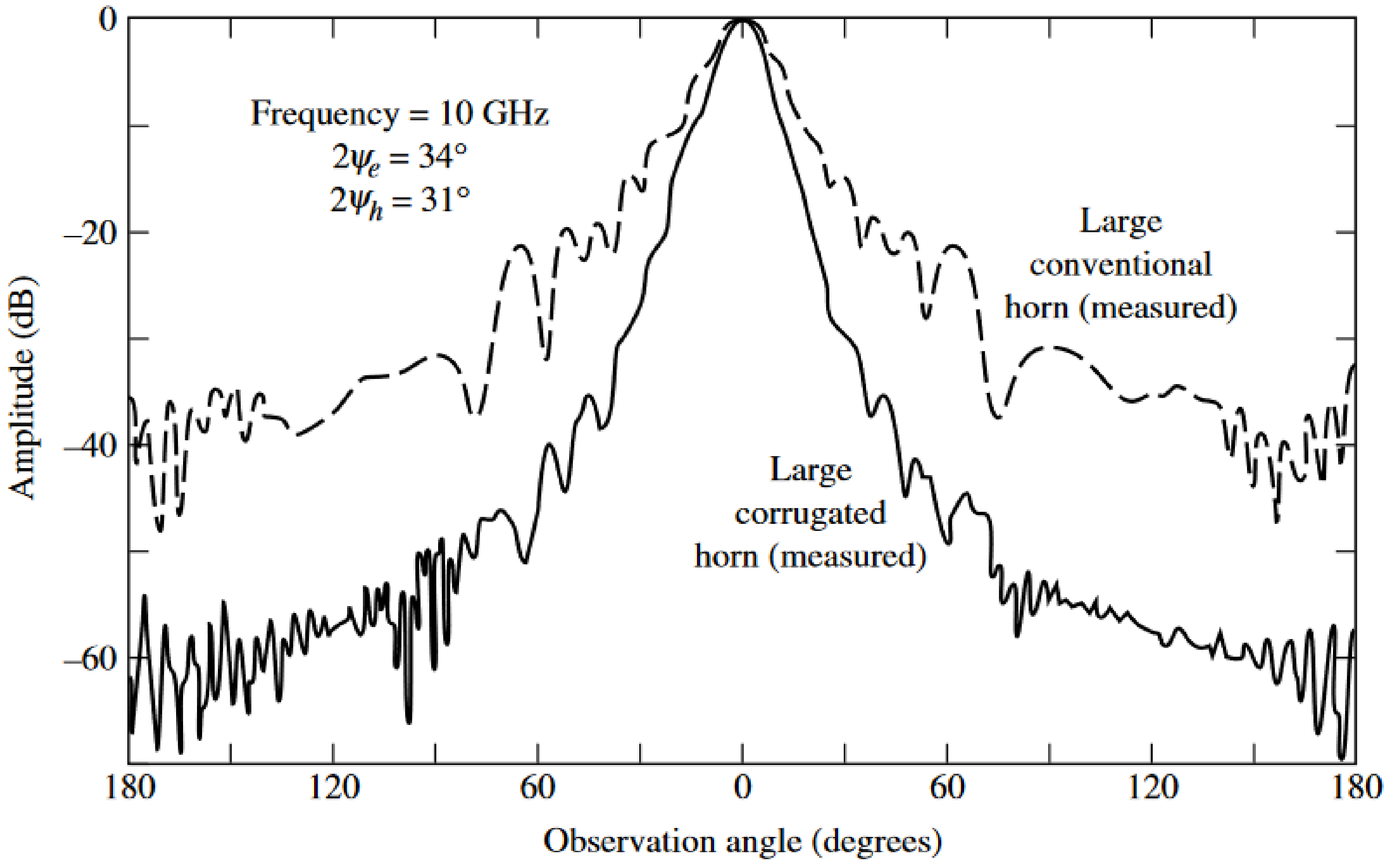
Horn Antenna

Corrugated Horn

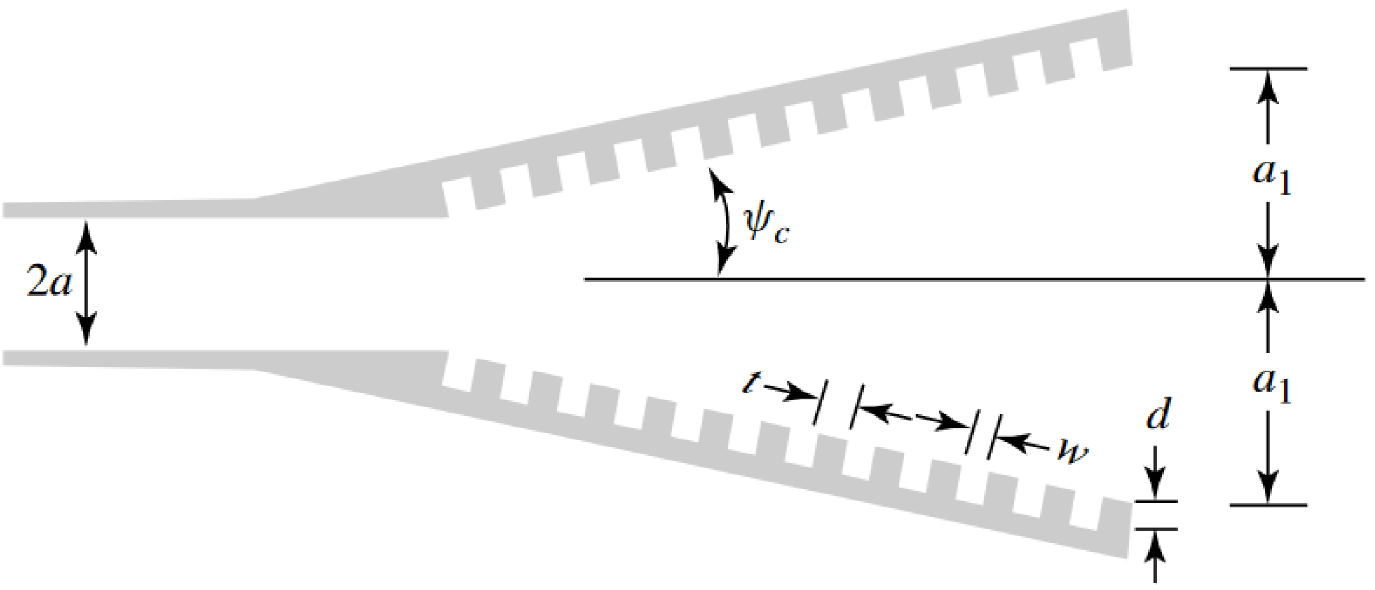
The large emphasis placed on horn antenna research in the 1960s was inspired by the need to reduce ***spillover efficiency*** and ***cross-polarization*** losses and increase *aperture efficiencies* of *large reflectors* used in radio astronomy and satellite communications. In the 1970s, high-efficiency and rotationally symmetric antennas were needed in microwave radiometry. Using conventional feeds, aperture efficiencies of 50–60% were obtained. However, efficiencies of the order of 75–80% can be obtained with improved feed systems utilizing corrugated horns.



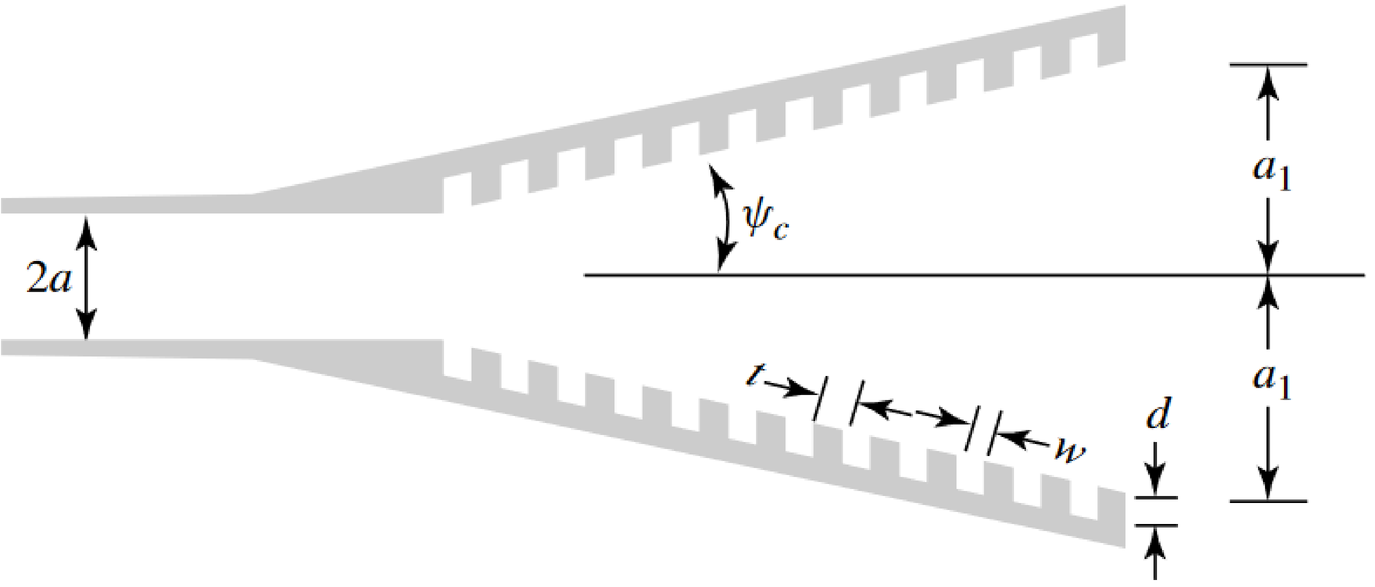
(a) *E*-plane ($2.96\lambda \times 2.96\lambda$)



Radiation characteristics of conventional (control), corrugated, and aperture-matched pyramidal horns



(a) Corrugations perpendicular to surface

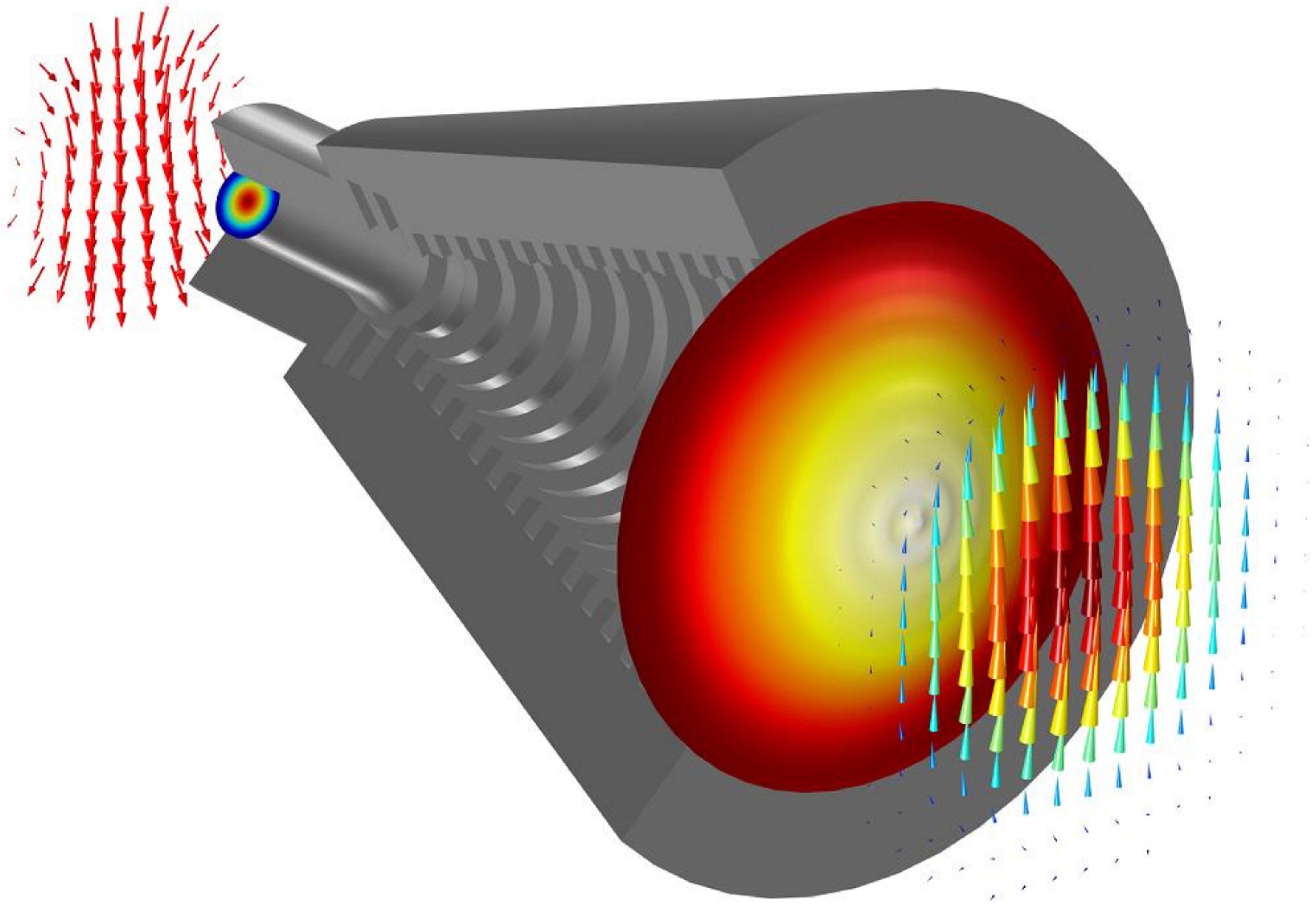


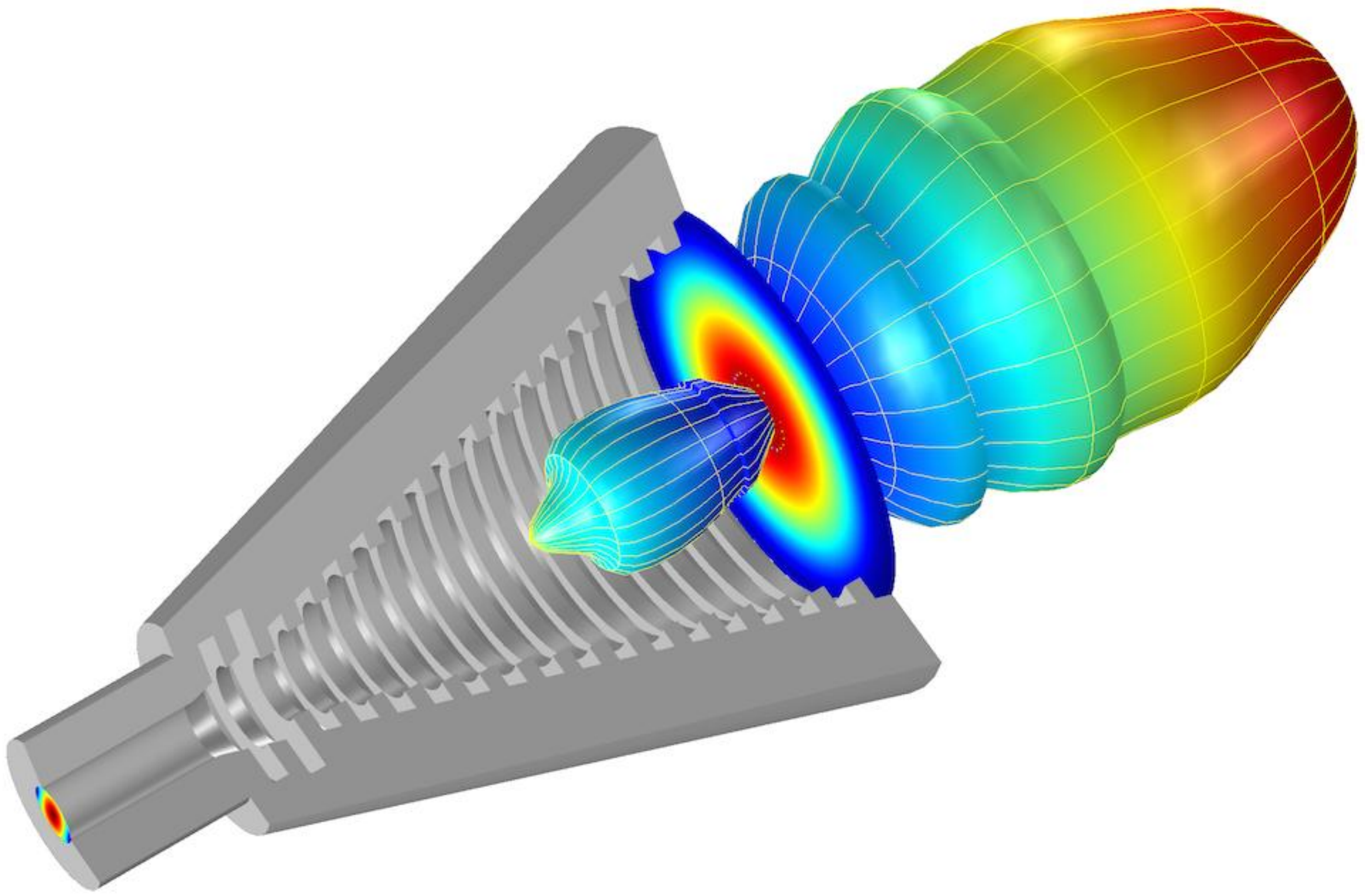
(b) Corrugations perpendicular to axis

Side view profiles of conical corrugated horns.

Horn Antenna

Corrugated Horn





Horn Antenna

Ridged Horn

A ridged horn antenna is a type of ***ultrawideband*** antenna that uses two parallel ridges inside the horn structure to improve performance over a wide frequency range, offering high gain, wide bandwidth, and stable performance for applications like EMC testing, radar, and communication systems. The ridges help to broaden the antenna's operational frequency band and provide *better impedance matching*.

Double-Ridged Horn

Quad-Ridged Horn

Antennas

Horn Antenna

Ridged Horn

Double-Ridged Horn Antenna (DRGH)

- Bandwidth: Good broadband operation, often in the range of 1-18 GHz.
- Design: Characterized by two ridges within the waveguide, which gradually increase the characteristic impedance to achieve broadband matching.
- Applications: Suitable for feeding high-gain reflectors, imaging, and for general EMC measurements

Quad-Ridged Horn Antenna (QRGH)

- Bandwidth: Provides even wider operational bandwidth than double-ridged horns.
- Sidelobe Levels: Offers lower sidelobe levels (SLL) due to the additional ridges, improving pattern performance.
- Gain: Achieves high gain, typically 10–20 dBi.
- Power Handling: Capable of handling higher RF power, making them suitable for transmitting applications.
- Dual Polarization: Often designed for dual-polarization and can function as a primary radiator for high-gain, long focal length reflectors.
- Applications: Excellent for reflector feeding, surveillance, direction-finding, and other demanding applications where enhanced performance is needed.

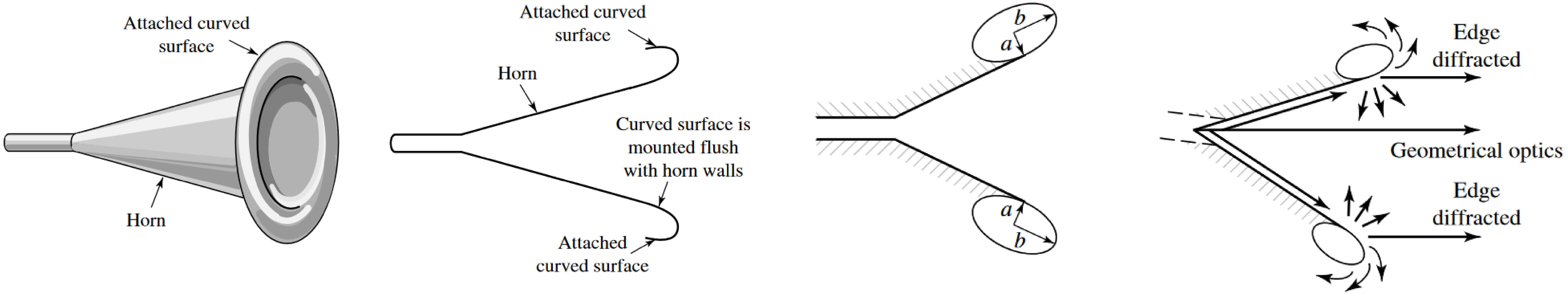
Key Comparison Points

- Bandwidth: QRGHs have a wider bandwidth.
- Pattern & Sidelobes: QRGHs typically have lower sidelobe levels.
- Power: QRGHs can handle higher power.
- Application: QRGHs are preferred for more advanced and demanding roles, while DRGHs are more general-purpose broadband antennas.

Horn Antenna

Aperture-Matched Horn

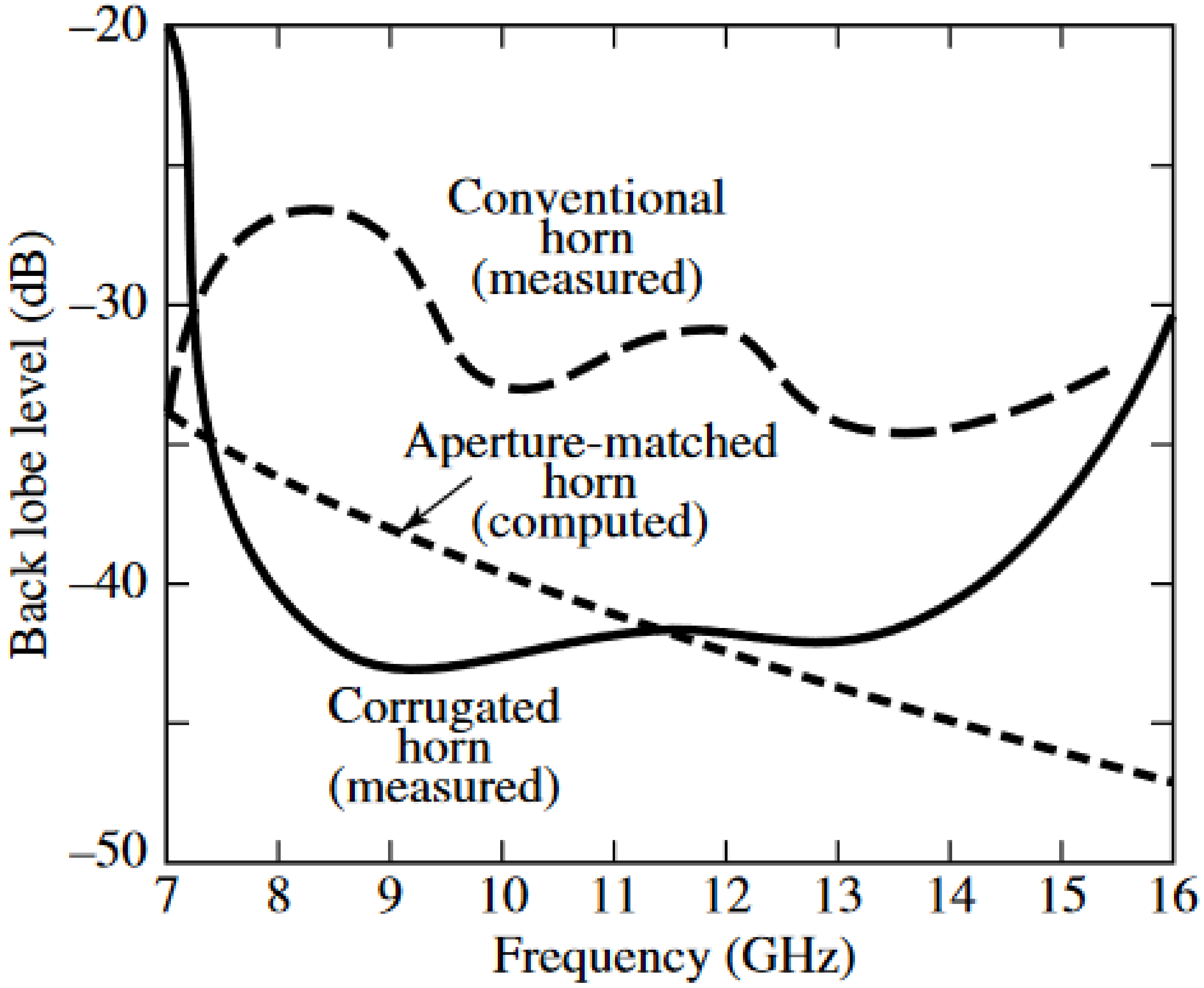
An aperture-matched horn provides significantly better performance than an ordinary horn (interms of pattern, impedance, and frequency characteristics). The main modification to the ordinary (conventional) horn, consists of the attachment of ***curved-surface*** sections to the outside of the aperture edges, which reduces the ***diffractions that occur at the sharp edges*** of the aperture and provides smooth matching sections between the horn modes and the free-space radiation. The radii of curvature of the curved surfaces used in experimental models ranged over $1.69\lambda \leq a \leq 8.47\lambda$ with $a = b$ and $b = 2a$. Good results can be obtained by using circular cylindrical surfaces with $2.5\lambda \leq a \leq 5\lambda$.



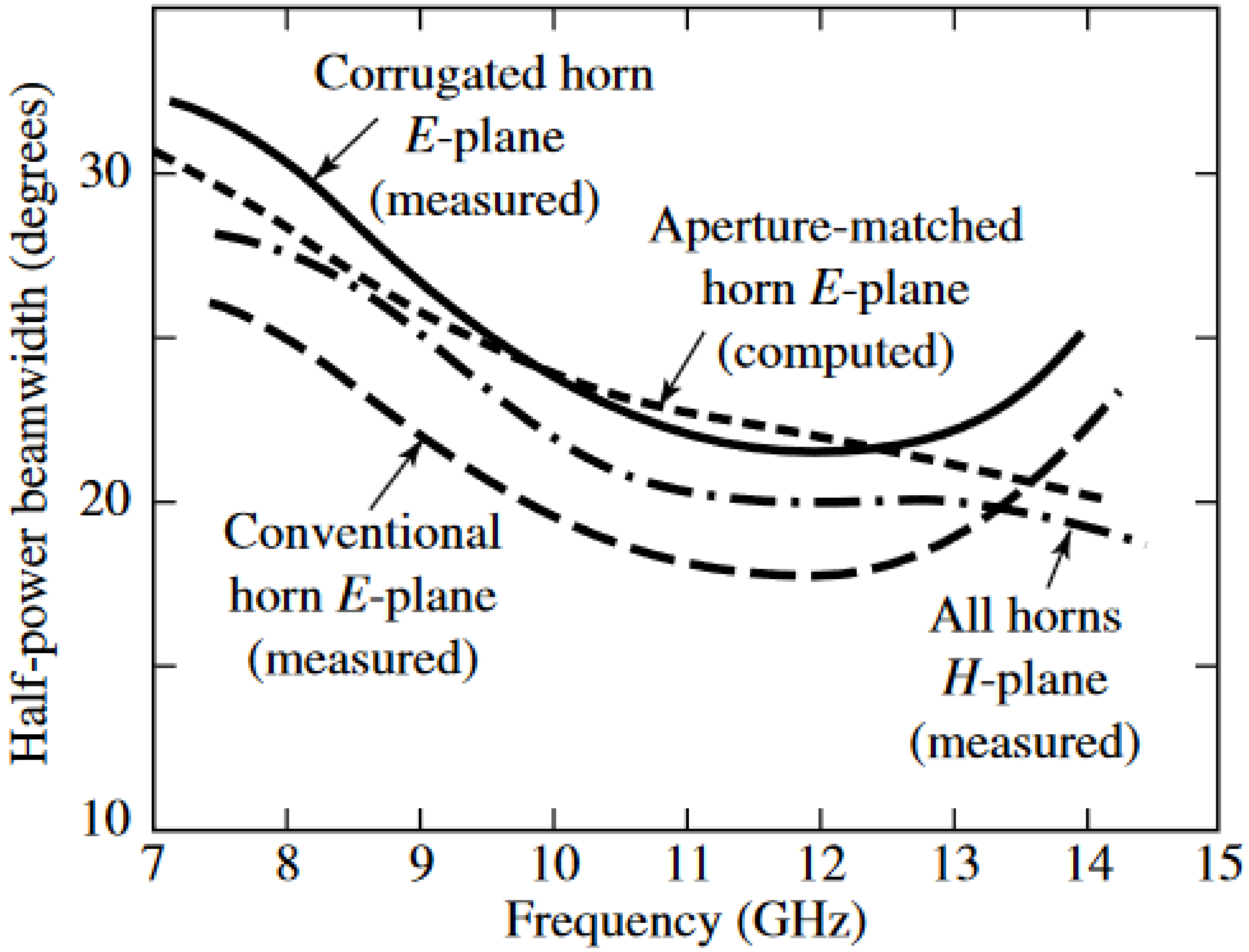
Geometry and diffraction mechanism of an aperture-matched horn.

Horn Antenna

Comparison



(c) E-plane back lobe level ($2.96\lambda \times 2.96\lambda$)



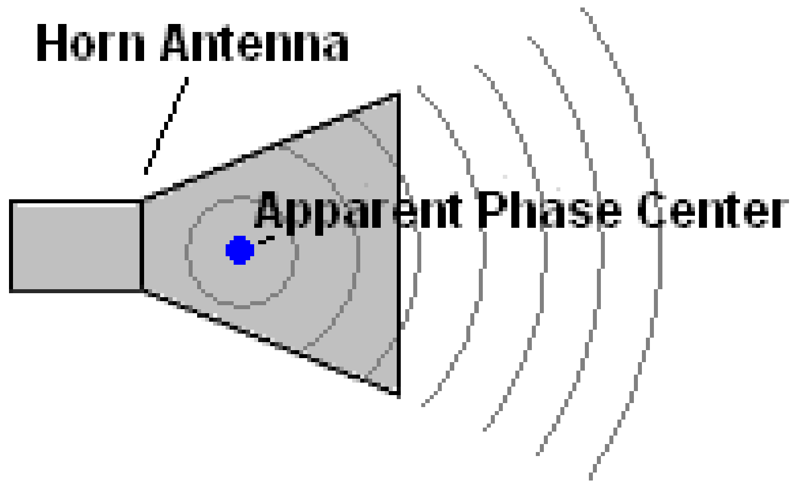
(d) E-plane half-power beamwidth ($2.96\lambda \times 2.96\lambda$)

Radiation characteristics of conventional (control), corrugated, and aperture-matched pyramidal horns

Horn Antenna

Phase Center

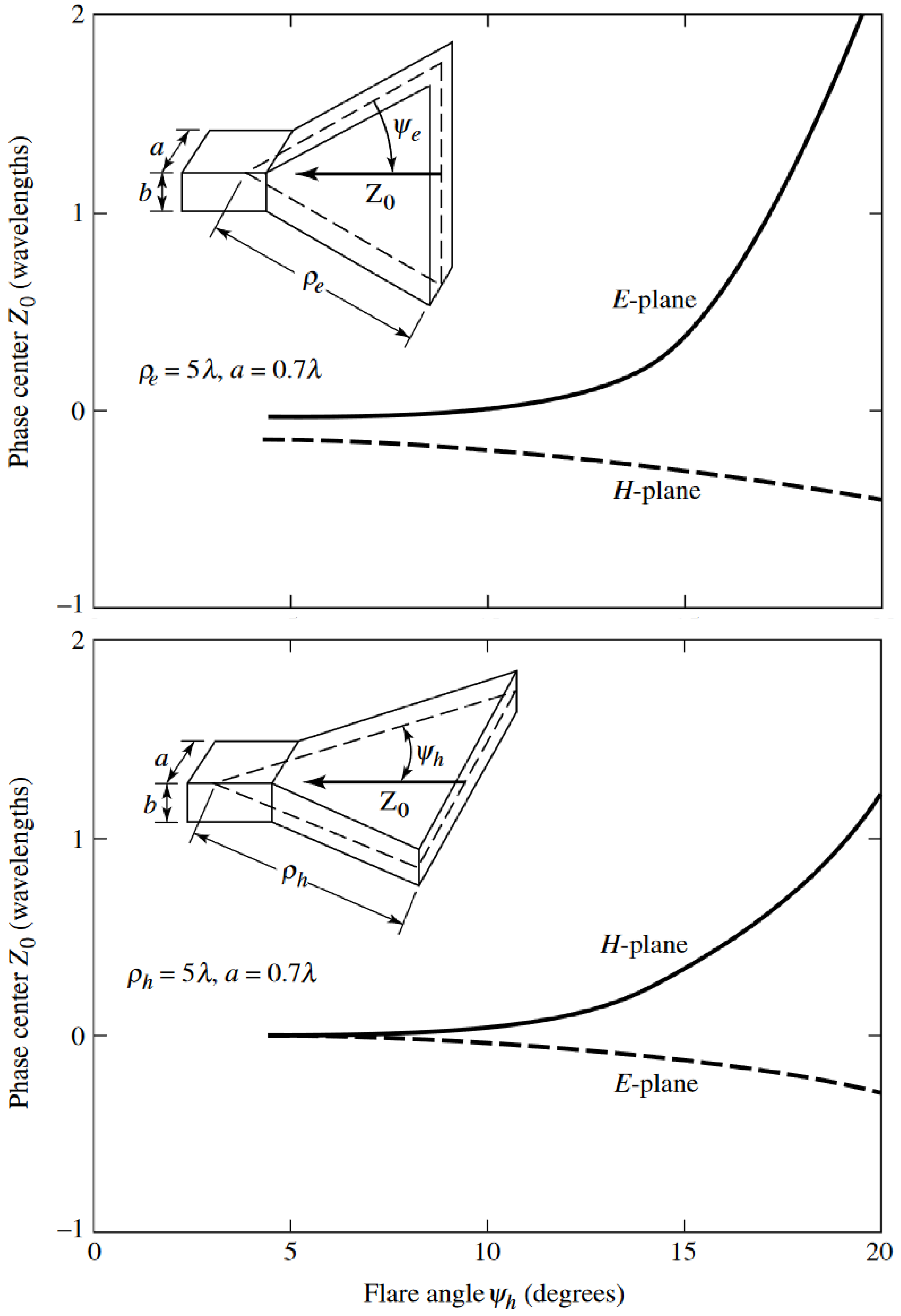
In antenna design theory, the ***phase center*** is the point from which the electromagnetic radiation spreads spherically outward, with the phase of the signal being equal at any point on the sphere. Apparent phase center is used to describe the phase center in a limited section of the radiation pattern.



Each far-zone field component radiated by an antenna can be written, in general, as

$$\mathbf{E}_u = \hat{\mathbf{u}} E(\theta, \phi) e^{j\psi(\theta,\phi)} \frac{e^{-jkr}}{r}$$

In navigation, tracking, homing, landing, and other aircraft and aerospace systems it is usually desirable to assign to the antenna a reference point such that for a given frequency, $\psi(\theta, \varphi)$ in the equation is independent of θ and φ (i.e., $\psi(\theta, \varphi)$ = constant). The reference point which makes $\psi(\theta, \varphi)$ independent of θ and φ is known as the phase center of the antenna. When referenced to the phase center, the fields radiated by the antenna are spherical waves with ideal spherical wave fronts or equi-phase surfaces. Therefore a phase center is a reference point from which radiation is said to emanate, and radiated fields measured on the surface of a sphere whose center coincides with the phase center have the same phase.



Phase center location, as a function of flare angle, for ***E***- and ***H*** ***-plane sectoral horns***

Antennas

Thank you!